Graduate Course Description

Course Title: Introduction to Hamiltonian Dynamics

Instructor: R. de la Llave **Semester: Fall 2105** Fall 15

Time and Place: TR 1:35pm-2:55pm Skiles 246

Brief description: Hamiltonian mechanics is the mathematical description of many physical phenomena.

Mechanics has served as the origin of many interesting mathematical developments including differential equations, topology (analysis situs) symplectic geometry, calculus of variations and ergodic theory. It is illumination to see that many different deep mathematical structures can illuminate some real problems.

We plan to introduce several topics that are of current interest in research as well as different techniques and points of view.

There will be assignments that will be either computational or theoretical, depending on the taste of the students. The handouts will not be graded.

Students will have to write a project or present a paper negotiated with the instructor.

The final grade will be based on delivering the project/presentation and handing in the assignments.

The prerequisites will be multivarable calculus and some basic elements of real analysis (Fourier series, contractions). (See e.g the first chapters of the book of Fassano-Marmi below).

If needed, we will provide some extra lectures for the students to catch up with the prerequisites.

Having some programming experience in either Matlab, C, Fortran, Python is not needed, but could be useful to gather intuition.

- Differential geometry.
  - Differential forms and Cartan calculus.
  - Basic facts about symplectic geometry.
- Mathematical formulations of mechanics.
  - Newtonian formulation.
  - Lagrangian formulation.
  - Hamiltonian formulation.
  - The origins of Hamiltonian formalism. Relation to optics.
- Examples (a few will be selected for coverage in class. A detailed study by the students will be encouraged.)
  - Kepler problem.
  - 3 and N body problems in Newton's gravitation.
  - Charged particles in electromagnetic fields.
  - Spherical pendulum.

  - Oscillators.
  - Integrable systems (Toda Lattice, Calogero system).
  - Numerical algorithms for eigenvalues as Hamiltonian systems.
  - Geodesic flows.
  - Hydrodynamics as a Hamiltonian system (formally)
- Transformation theory
  - Normal forms.
  - Generating functions.
  - Lie series.
  - Deformation theory.
- Perturbation theory.
  - Perturbation theory for periodic orbits.
  - Lindstedt series for quasi-periodic orbits.
  - Poincaré's proof of the stable manifold theorem.
  - Melnikov method.
  - Canonical perturbation theory.
  - Numerical implementation of Lindstedt series. (optional)
  - Numerical calculation of invariant manifolds with Poincaré method.(optional)
- KAM theorem on persistence of smooth tori.
  - Automatic reducibility
  - The Kolmogorov proof.
  - The Arnol'd proof. (Presumably only a sketch)
  - Numerical implementations (optional).
- Variational theory.
  - Lagrangian submanifolds and their intersections.
  - Variational theory for periodic orbits.
  - Variational methods for quasi-periodic orbits.
  - Morse theory of geodesics.
  - Weak KAM theory.
  - Aubry-Mather theory.
- Other topics to be discussed if there is time or to be assigned as projects. The following are more tentative:
- Ergodic theory.
  - The origins of ergodic theory and the ergodic hypothesis.
  - Koopman formalism.
  - Von Neuman Ergodic theorem.
  - Birkhoff Ergodic theorem.
  - Mixing
  - Entropy and Shannon Mc Millan Breiman theorem.
  - Poincaré-Birkhoff fixed point theorem.
    - Birkhoff version.
    - Conley-Zehnder version.
    - Franks version.
    - Le Calvez version.
    - Poincaré version.

Geodesic flows. Anosov systems
Billiards
Non-existence of invariant tori.
Renormalization description of breakdown of tori.
Renormalization description of period doubling.
Systems of coupled oscillators.
Infinite dimensional Hamiltonian.
Nekhoroshev theorem

Prerequisites:

Textbook(s):

W. Thirring: A course in Mathematical Physics Vol I (Reccomended)
V. I. Arnol'd: Mathematical Methods of Classical Mechanics (Recommended)
V. I. Arnol'd: Geometric methods in the theory of differential equations (Reccomended)

D. Treschev, O. Zubelich: Introduction to the perturbation theory of Hamiltonian systems. The first 2 chapters are about topics we hope to cover. The other chapters could be the basis of independent reading.
Other readings

G. Gallavotti: The elements of mechanics. This book uses a very down to earth approach – no differential geometry – but is very good to learn how to analyze concrete examples and how to compute.

A. Fassano, S. Marmi: Analytical Mechanics. This book has many concrete calculations besides the formalism.

The 9 volumes of the Encyclopaedia of Mathematical Sciences devoted to dynamical systems also contain a wealth of modern material. They are designed to be read in almost any order and contain lots of pointers to modern literature.

R. Abraham and Marsden: Foundations of Mechanics is a good book but requieres careful selection of the chapters. They can be read in diferent orders.

M. Levi The mathematical mechanic (just for fun)