ON SEQUENCES OF ITERATES OF GENERALIZATION OF $ho$-CONTRACTIONS

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Given a number $\rho > 0$ an operator $T$ on a Hilbert space $\mathcal{H}$ is called a $\rho$-contraction if there exists a Hilbert space $\mathcal{K}$ containing $\mathcal{H}$ and a unitary operator $U \in L(\mathcal{K})$ such that

$$T^n h = \rho^n P h U^n h \text{ for } h \in \mathcal{H} \text{ and } n \in \mathbb{N}^* \ (1).$$

Thus, by the celebrated Nagy-Foias dilation result, 1-contractions are exactly usual contractions. In fact Nagy and Foias are “responsible” for the introduction (including the terminology) and fundamental properties of these operators (back in the sixties) motivated by a so-called “strange dilation result” of Berg saying (in the above terms) that 2-contractions are exactly the operators whose numerical radius ($w(T) := \sup_{\|x\|=1} |(Tx, x)|$) is equal to 1.

Eckstein (back in 1972) proved that, for any $\rho$-contraction $T$ on a Hilbert space $\mathcal{H}$ and any vector $h \in \mathcal{H}$, the sequence $\{\|T^n h\|\}$ is convergent (a fact which is elementary for ordinary contractions). We show that this remains true for a natural generalization of the class of $\rho$-contractions, which we call the class of $(\rho, N)\text{-contractions}$ (the definition is as above but with the equality (1) holding for $n \geq N$ where $N$ is a given natural integer). Our argument follows the lines of Mlak’s proof of Eckstein’s result, but is somewhat simplified by a study of coisometric $(\rho, N)\text{-dilations}$ of these operators, which seems to be of independent interest. Along the way we also point out that Gavruta’s example, disproving a conjecture of Berneeci-Foias according to which an operator $T$ such that $T^p$ is $\rho$-contraction might be a $\rho'$-contraction for some $\rho'(\geq \rho)$, extends to $(\rho, N)$-contractions. Namely, given any integer $p > 1$, there exists an operator $T$ such that $T^p = I$ which is not a $(\rho, N)$-contraction (for whatever choice of $\rho$ and $N$).

This is joint work with Dan Crăciunescu (West University Timişoara).