On the CLT for sample covariance matrices without independent structure in columns

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In multivariate statistical inference, many test statistics are defined by the eigenvalues of large sample covariance (SC) matrices

\[ M_n = Y Y^T, \quad Y = (y_1 \ y_2 \ \ldots \ y_m), \]

where \( \{y_\alpha\}_\alpha \) are independent random vectors in \( \mathbb{R}^n \). Under the assumption that all entries of \( Y \) are independent, the asymptotical spectral analysis of SC matrices has been actively developed since the celebrated work of Marčenko and Pastur (1967). Less is known about large SC matrices with dependent entries of \( y_\alpha \). In papers by Bai and Zhou, Pajor and Pastur, and Yaskov, there were considered limiting spectral distributions of SC matrices under some general dependence structures of \( y_\alpha \). The next natural step is to study asymptotical fluctuations of linear eigenvalue statistics \( \text{Tr} \varphi(M_n) \), where \( \varphi \) is a test function. Here we consider SC matrices of the form

\[ M_n = \sum_{\alpha=1}^{m} \tau_\alpha y_\alpha y_\alpha^T, \]

where \( \{\tau_\alpha\}_\alpha \) are real numbers and \( \{y_\alpha\}_\alpha \) are i.i.d. normalized isotropic random vectors in \( \mathbb{R}^n \) satisfying certain moment conditions. We show that in the limit \( m, n \to \infty \ m/n \to c \in (0, \infty) \), the centered linear eigenvalue statistics converge in distribution to a Gaussian random variable.