KEY (E)

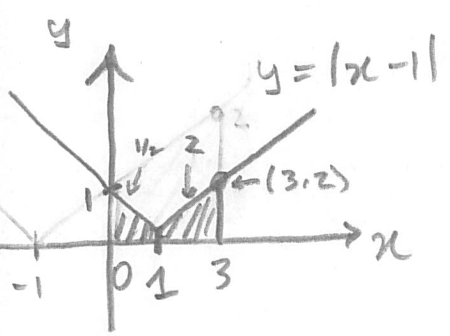
1. Short answer section: fill in the equality with the correct expression. (5 pts. each)

(a) If $F'(x) = f(x)$, then $\int f(g(x))g'(x)\,dx = \boxed{F(g(x)) + C}$

(b) From the FTC we have $\frac{d}{dx}\int_a^{g(x)} f(t)\,dt = \boxed{f(g(x)) \cdot g'(x)}$

2. Evaluate the integral using any method, but clearly show your work for full credit. *Hint: draw a picture.* (10 pts.)

$$\int_0^3 |x-1|\,dx$$



geometry $\frac{1}{2} + 2 = \boxed{5/2}$

$$\int_0^3 |x-1|\,dx = \int_0^1 -(x-1)\,dx + \int_1^3 (x-1)\,dx$$

$$= -\tfrac{1}{2}x^2 + x\Big|_0^1 + \tfrac{1}{2}x^2 - x\Big|_1^3$$

$$= \left(-\tfrac{1}{2} + 1 - 0\right) + \left(\tfrac{1}{2}9 + 3 - \tfrac{1}{2} + 1\right)$$

$$= \tfrac{9}{2} - 3 + 1 = \tfrac{9}{2} - \tfrac{4}{2} = \boxed{5/2}$$

3. Evaluate the integral. (14 pts.)

$$\int_1^e \frac{\ln x}{x}\,dx$$

$\boxed{\begin{array}{l} u = \ln x \\ du = \frac{1}{x}\,dx \end{array}}$

$$= \int_*^* u\,du = \frac{1}{2}u^2\Big|_*^*$$

$$= \frac{1}{2}(\ln x)^2\Big|_1^e$$

$$= \frac{1}{2}(\ln e)^2 - \frac{1}{2}(\ln(1))^2$$

$$= \frac{1}{2}(1)^2 - 0$$

$$= \boxed{1/2}$$

4. Solve the separable differential equation. (14 pts.)

$$\frac{dy}{dx} = \frac{y\sec^2(\sqrt{x})}{\sqrt{x}}$$

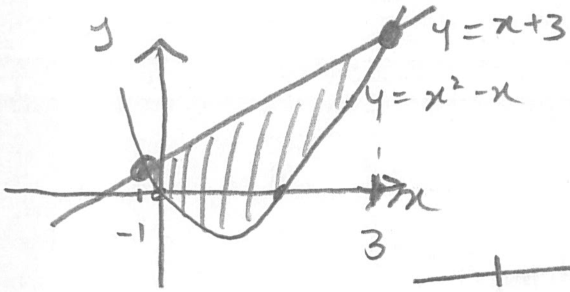
$$\int \frac{1}{y}\,dy = \int \sec^2(\sqrt{x}) \cdot \frac{1}{\sqrt{x}}\,dx$$

$$\ln(y) = 2\tan(\sqrt{x}) + C$$

$$y = e^{2\tan\sqrt{x} + C} = \boxed{C \cdot e^{2\tan\sqrt{x}}}$$

either ok

5. Find the area bounded by the curves $y = x^2 - x$ and $y = x + 3$. Show all your work for credit. (14 pts.)



$x(x-1)$

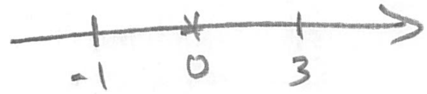
Find int. pts.

$x^2 - x = x + 3$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow (x-3)(x+1) = 0$

$x = 3, -1$



quad $0^2 - 0 = 0$ BOT

lin $0 + 3 = 3$ TOP

$$\text{Area} = \int_{-1}^{3} [(x+3) - (x^2 - x)]\,dx = \int_{-1}^{3} (-x^2 + 2x + 3)\,dx$$

$$= -\tfrac{1}{3}x^3 + x^2 + 3x \Big|_{-1}^{3} = \left(-\tfrac{1}{3}3^3 + 9 + 9\right) - \left(-\tfrac{1}{3}(-1) + 1 - 3\right)$$

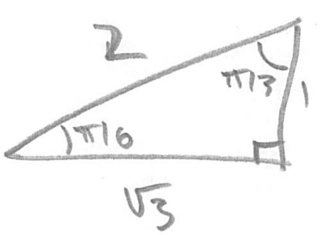
$$= 9 - \tfrac{1}{3} + 2 = 11 - \tfrac{1}{3} = \boxed{32/3}$$

6. Let $y = \int_0^{\sec x} \sin(\pi t^2)\,dt$. Find the value of $\frac{dy}{dx}$ when $x = \pi/3$. (14 pts.)

$$\frac{dy}{dx} = \sin(\pi \cdot \sec^2 x) \cdot \sec(x)\tan(x)$$

$$\frac{dy}{dx}\Big|_{x=\pi/3} = \sin(\pi \cdot \sec^2(\pi/3)) \sec(\pi/3)\tan(\pi/3)$$

$$= \sin(\pi(2)^2) \cdot 2 \cdot \sqrt{3}$$



$$= \sin(4\pi) \cdot 2\sqrt{3}$$

$$= \boxed{0}$$

7. Evaluate the limit using any method. Clearly indicate your work for full credit. (10 pts.)

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{3}{n}\left(2\left(1+\frac{3i}{n}\right)^2-1\right)$$

$$= \int_1^4 (2x^2-1)\,dx = \frac{2}{3}x^3 - x \Big|_1^4$$

$$= \left(\frac{2}{3}\cdot 4^3 - 4\right) - \left(\frac{2}{3}-1\right)$$

$$= \frac{128-12-2+3}{3}$$

$$= \boxed{\frac{117}{3}}$$

8. Find a closed formula for L_n the left-endpoint Riemann sum for the function $y = x^2+1$ over the interval $[0,2]$. (14 pts.)

$\Delta x = 2/n$

$x_i = 2i/n$

$f(x) = x^2+1$

$L_n = \sum_{i=0}^{n-1} \Delta x \cdot f(x_i)$

$$L_n = \sum_{i=0}^{n-1} \Delta x \cdot f(x_i) = \sum_{i=0}^{n-1} \frac{2}{n}\left(\left(\frac{2i}{n}\right)^2+1\right)$$

$$= \frac{8}{n^3}\cdot\sum_{i=0}^{n-1} i^2 + \frac{2}{n}\sum_{i=0}^{n-1} 1$$

$$= \frac{8}{n^3}\cdot\frac{n(n-1)(2n-1)}{6} + 2$$

$$\lim_{n\to\infty} L_n = 8\cdot\frac{2}{6} + 2 = \frac{8}{3}+2 = \frac{8+6}{3} = \boxed{14/3}$$

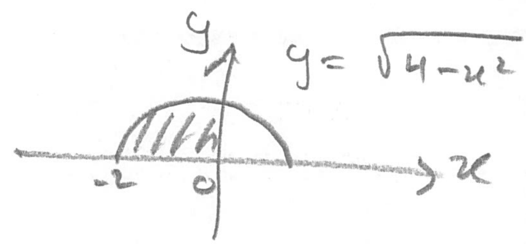
Check $\int_0^2 (x^2+1)\,dx = \frac{1}{3}x^3 + x\Big|_0^2 = \frac{1}{3}\cdot 8 + 2 - 0 = 8/3 + 2 = \boxed{14/3}$

Bonus: *(up to +5pts.)* Find the average value of $y = \sqrt{4 - x^2}$ over the interval $[0, 2]$.

$$\frac{1}{2-0} \int_0^2 \sqrt{4 - x^2}\, dx$$

$$= \frac{1}{2} \cdot 4\pi$$

$$= \boxed{\pi/2}$$



$$\frac{1}{4} \pi (2)^2 = \frac{4\pi}{4} = \pi$$

$$\frac{4\pi}{2} = 2\pi,$$