Quiz 3-E

1. Determine if each series converges or diverges. Justify completely by *(a)* clearly stating the test you used, *(b)* showing any necessary work to justify the answer, and *(c)* giving a summary sentence which explains how the test was used.

   (a) (9 points) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(\sqrt{n}+1)^2}$

$\boxed{\begin{array}{l} u = \sqrt{x}+1 \\ du = \frac{1}{2\sqrt{x}}\,dx \end{array}}$

Integral test $\displaystyle\int_1^{\infty} \frac{1}{\sqrt{x}(\sqrt{x}+1)^2}\,dx = \int_2^{\infty} \frac{2}{u^2}\,du < \infty$

by p-test w/ $p = 2 > 1$

So the series $\sum a_n$ converges by the integral test.

DCT $\displaystyle a_n = \frac{1}{\sqrt{n}(\sqrt{n}+1)^2} \leq \frac{1}{\sqrt{n}(\sqrt{n})^2} = \frac{1}{n^{3/2}} = b_n$ & $\sum b_n$ converges by p-test w/ $p = 3/2 > 1$.

So by DCT the series $\sum a_n$ $\boxed{\text{converges}}$

   (b) (9 points) $\displaystyle\sum_{n=2}^{\infty} \frac{n+2}{n^2-1}$

LCT w/ $b_n = 1/n$

$\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\left(n+2/n^2-1\right)}{1/n}$

$\displaystyle = \lim_{n\to\infty} \frac{(n+2)(n)}{n^2-1} = \lim_{n\to\infty} \frac{n^2+2n}{n^2-1} = 1 = L$

Since harmonic series $\sum \frac{1}{n}$ diverges and $0 < L < \infty$, by LCT the series $\sum a_n$ also $\boxed{\text{diverges}}$

Or

DCT w/ $b_n = 1/n$

$\displaystyle a_n = \frac{n+2}{n^2-1} \geq \frac{n}{n^2-1} \geq \frac{n}{n^2} = \frac{1}{n} = b_n$

Since $a_n \geq b_n$ and $\sum b_n$ diverges (harmonic series w/ $p=1$)

The series $\sum a_n$ $\boxed{\text{diverges}}$ by DCT.

2. (a) (2 points) State the geometric series formula.

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r} \quad \text{if} \quad |r|<1$$

(b) (10 points) Find the **value** of the geometric series. Clearly show each step of your work using correct notation for full credit.

$$\sum_{n=1}^{\infty} \frac{4}{5^n}$$

$$4 \cdot \sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^n = 4 \cdot \frac{1}{5}\left(\frac{1}{1-1/5}\right) = \frac{4}{5} \cdot \left(\frac{1}{4/5}\right) = \boxed{1}$$

or

$$4 \sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^n = 4\left(\frac{1}{1-1/5} - 1\right) = 4 \cdot \left(\frac{5}{4} - 1\right) = 4 \cdot \frac{1}{4} = \boxed{1}$$