Quiz 3-G

1. Determine if each series converges or diverges. Justify completely by *(a)* clearly stating the test you used, *(b)* showing any necessary work to justify the answer, and *(c)* giving a summary sentence which explains how the test was used.

(a) (9 points) $\sum_{n=1}^{\infty} \frac{1}{n(\ln n+1)^2}$

Integral test $\int_1^{\infty} \frac{1}{x(\ln x+1)^2}\,dx = \int_1^{\infty} \frac{1}{u^2}\,du < \infty$

$\boxed{\begin{aligned} u &= \ln x+1 \\ du &= \tfrac{1}{x}\,dx \end{aligned}}$

by p-test w/ $p=2>1$

So by integral test the series $\sum a_n$ $\boxed{\text{converges}}$

(b) (9 points) $\sum_{n=1}^{\infty} \frac{n-3}{n^3+n}$

LCT w/ $b_n = \frac{1}{n^2}$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\left(\frac{n-3}{n^3+n}\right)}{1/n^2}$$

$$= \lim_{n\to\infty} \frac{(n-3)(n^2)}{n^3+n}$$

$$= \lim_{n\to\infty} \frac{n^3-3n^2}{n^3+n} = 1 = L$$

Since $0<L<1$ and $\sum \frac{1}{n^2}$ converges by p-test w/ $p=2>1$, The series $\sum a_n$ also $\boxed{\text{Converges}}$ by LCT.

OR

DCT w/ $b_n = 1/n^2$

Since $a_n = \frac{n-3}{n^3+n} \leq \frac{n}{n^3+n} \leq \frac{n}{n^3} = \frac{1}{n^2} = b_n$

And $\sum \frac{1}{n^2}$ converges by p-test w/ $p=2>1$,

by DCT The series $\sum a_n$ $\boxed{\text{converges}}$

2. (a) (2 points) State the geometric series formula.

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r} \quad \text{if } |r|<1.$$

(b) (10 points) Find the **value** of the geometric series. Clearly show each step of your work using correct notation for full credit.

$$\sum_{n=1}^{\infty} \frac{3}{4^n}$$

$$3 \cdot \sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n = 3 \cdot \frac{1}{4} \left(\frac{1}{1-1/4}\right)$$

$$= \frac{3}{4} \cdot \left(\frac{1}{3/4}\right) = \boxed{1}$$

OR

$$3 \cdot \sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n = 3 \cdot \left(\frac{1}{1-1/4} - 1\right) = 3 \cdot \left(\frac{4}{3} - 1\right) = 3 \cdot \frac{1}{3} = \boxed{1}$$