Quiz 4 - E

1. Determine if each series converges or diverges. Justify completely by *(a)* clearly stating the test you used, *(b)* showing any necessary work to justify the answer, and *(c)* giving a summary sentence which explains how the test was used.

   (a) (9 points) $\sum_{n=1}^{\infty} \frac{n^3}{(2n)!}$

ratio test

$a_n = \frac{n^3}{(2n)!}$

$a_{n+1} = \frac{(n+1)^3}{(2n+2)!}$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)^3}{(2n+2)!} \cdot \frac{(2n)!}{n^3}$$

$$= \lim_{n\to\infty} \frac{1}{(2n+2)(2n+1)} \cdot \frac{(n+1)^3}{n^3} = 0$$

$\Rightarrow 0 = \rho < 1$

Since $\rho<1$, by the ratio test the series $\sum a_n$ $\boxed{\text{converges}}$

   (b) (9 points) $\sum_{n=1}^{\infty} \frac{3}{(2n+1)^n}$

root test

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \frac{3^{1/n}}{2n+1} = 0 \ (= 1/\text{big}) \quad \text{So } \rho = 0.$$

Since $\rho<1$, by the root test the series

$\boxed{\text{converges}}$

2. (a) (3 points) Find the limit, showing all steps and justifying with L'Hopital's Rule, or state the formula you used.

$$\lim_{n\to\infty}\left(1-\frac{3}{n}\right)^n = e^{-3}$$

by formula $\lim_{n\to\infty}\left(1+\frac{p}{n}\right)^{qn} = e^{pq}$

(b) (9 points) Determine if the given series converges or diverges. Justify completely by *(a)* clearly stating the test you used, *(b)* showing any necessary work to justify the answer, and *(c)* giving a summary sentence which explains how the test was used.

$$\sum_{n=1}^{\infty}\left(1-\frac{3}{n}\right)^n$$

By the divergence test (aka n-th term test) the series $\sum a_n$ diverges since $\lim_{n\to\infty} a_n \neq 0$.