Quiz 4-G

1. Determine if each series converges or diverges. Justify completely by *(a)* clearly stating the test you used, *(b)* showing any necessary work to justify the answer, and *(c)* giving a summary sentence which explains how the test was used.

   (a) (9 points) $\sum_{n=1}^{\infty} \left(\frac{4n+3}{3n+2}\right)^n$

root test

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \frac{4n+3}{3n+2} = \frac{4}{3} = \rho > 1$$

Since $\rho > 1$, by the root test the series $\boxed{\text{diverges}}$

   (b) (9 points) $\sum_{n=1}^{\infty} \frac{(2n)^3}{n!}$

ratio test

$$a_n = \frac{(2n)^3}{n!}$$

$$a_{n+1} = \frac{(2n+2)^3}{(n+1)!}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(2n+2)^3}{(n+1)!} \cdot \frac{n!}{(2n)^3}$$

$$= \lim_{n\to\infty} \frac{1}{n+1} \cdot \frac{(2n+2)^3}{(2n)^3} = 0 = \rho < 1$$

Since $\rho < 1$, by the ratio test the series $\boxed{\text{converges}}$

2. (a) (3 points) Find the limit, showing all steps and justifying with L'Hopital's Rule, or state the formula you used.

$$\lim_{n\to\infty}\left(1+\frac{1}{2n}\right)^n = e^{1/2}$$

by formula $\lim_{n\to\infty}\left(1+\frac{p}{n}\right)^{qn} = e^{pq}$

(b) (9 points) Determine if the given series converges or diverges. Justify completely by *(a)* clearly stating the test you used, *(b)* showing any necessary work to justify the answer, and *(c)* giving a summary sentence which explains how the test was used.

$$\sum_{n=1}^{\infty}\left(1+\frac{1}{2n}\right)^n$$

The Series $\sum a_n$ $\boxed{\text{diverges}}$

by the divergence test

(aka n-th term test)

since $\lim_{n\to\infty} a_n \neq 0$.