Quiz 5 - E

1. (9 points) Determine if the series converges or diverges. Show all work for credit.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$$

converges by alt. series test

Since $a_n = \frac{1}{\sqrt{n+1}}$ dec ✓

pos ✓

$a_n \to 0$ ✓

2. (9 points) Determine if the series converges absolutely, converges conditionally, or diverges. Show all work for credit.

$$\sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{n!}$$

absolute convergence?

Check Σa_n converges

ratio test

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{3^{n+1}/(n+1)!}{3^n/n!} = \lim_{n\to\infty} \frac{3^{n+1}}{3^n} \cdot \frac{n!}{(n+1)!}$$

$$= \lim_{n\to\infty} \frac{3}{n+1} = 0 = \rho$$

Since $\rho < 1$ the series Σa_n converges.

So $\Sigma (-1)^n a_n$ $\boxed{\text{converges absolutely}}$

3. (12 points) Find the radius and interval of convergence of the given power series. *Hint: Be sure to check each of the endpoints individually.*

$$\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n\, n^2}\, x^n$$

ratio test

$$\lim_{n\to\infty} \frac{a_{n+1}x^{n+1}}{a_n x^n} = \lim_{n\to\infty} \frac{2^{n+2}\, x^{n+1}}{3^{n+1}(n+1)^2} \cdot \frac{3^n\, n^2}{2^{n+1}\, x^n}$$

L'Hop twice

$$= \lim_{n\to\infty} \frac{2x}{3} \cdot \frac{n^2}{(n+1)^2} = \frac{2x}{3} = \rho$$

Set $|\rho|<1$ Solve $\left|\frac{2x}{3}\right|<1 \iff |x|<3/2$

$x \in (-3/2, 3/2)$.

Check endpoints.

$x = -3/2$ $\quad \sum \frac{2^{n+1}}{3^n n^2} \cdot \left(\frac{-3}{2}\right)^n = 2 \cdot \sum \frac{(-1)^n}{n^2}$ converges by alt. series test

$x = 3/2$ $\quad \sum \frac{2^{n+1}}{3^n \cdot n^2} \left(\frac{3}{2}\right)^n = 2 \cdot \sum \frac{1}{n^2}$ converges by p-test w/ $p=2>1$

So $\boxed{\text{interval } [-3/2, 3/2] \text{ \& radius } R = 3/2}$