Quiz 5 - G

1. (9 points) Determine if the series converges or diverges. Show all work for credit.

$$\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt{n}}{n^2+1}$$

converges by alternating series test

$a_n = \frac{\sqrt{n}}{n^2+1}$

a_n pos ✓
dec ✓
$a_n \to 0$ ✓

2. (9 points) Determine if the series converges absolutely, converges conditionally, or diverges. Show all work for credit.

$$\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt{n}}{(n+1)!}$$

Check absolute convergence

ratio test

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\sqrt{n+1}}{(n+1)!}\cdot\frac{n!}{\sqrt{n}}$$

$$= \lim_{n\to\infty}\frac{1}{n+1}\cdot\sqrt{\frac{n+1}{n}} = 0\cdot 1 = 0 = \rho$$

Since $\rho < 1$, the series $\sum a_n$ converges.

So $\sum(-1)^n a_n$ $\boxed{\text{Converges absolutely}}$

3. (12 points) Find the radius and interval of convergence of the given power series. *Hint: Be sure to check each of the endpoints individually.*

$$\sum_{n=1}^{\infty} \frac{2^n}{3^{n+1}\sqrt{n}} x^n$$

ratio test

$$\lim_{n\to\infty} \frac{a_{n+1}x^{n+1}}{a_n x^n} = \lim_{n\to\infty} \frac{2^{\cancel{n+1}}x^{\cancel{n+1}}}{3^{\cancel{n+2}}\sqrt{n+1}} \cdot \frac{\cancel{3^{n+1}}\sqrt{n}}{\cancel{2^n}\cdot\cancel{x^n}}$$

$$= \lim_{n\to\infty} \frac{2}{3}x \cdot \frac{\sqrt{n}}{\sqrt{n+1}} = \lim \frac{2}{3}x\cdot\sqrt{\frac{n}{n+1}} = \frac{2}{3}x\cdot 1 = \rho$$

Set $|\rho|<1$ & solve $\left|\frac{2}{3}x\right|<1 \iff |x|<3/2$

So $x \in (-3/2, 3/2)$.

Check endpoints.

$x = -3/2$ $\quad \sum \frac{2^n}{3^{n+1}\sqrt{n}}\left(\frac{-3}{2}\right)^n = \sum \frac{(-1)^n}{\sqrt{n}}$ converges by alt. series test

$x = 3/2$ $\quad \sum \frac{2^n}{3^{n+1}\sqrt{n}}\left(\frac{3}{2}\right)^n = \sum \frac{1}{\sqrt{n}}$ diverges by p-test w/ $p = 1/2 \leq 1$.

So $\boxed{\text{interval } [-3/2, 3/2) \text{ and radius } 3/2 = R}$