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1. **True or False?** Determine if each statement below is always true or sometimes false. Circle your answer to each statement.

   (a) (2 points) TRUE $\boxed{\text{FALSE}}$ To evaluate $\int x \tan^{-1}(x)\, dx$ by parts, choose $u = x$ and $dv = \tan^{-1}(x)\, dx$.

   (b) (2 points) $\boxed{\text{TRUE}}$ FALSE If f is a continuous function, then the function $F(x) = \int_a^x f(t)dt$ is an anti-derivative of f.

   (c) (2 points) TRUE $\boxed{\text{FALSE}}$ The goal of integration by parts is to go from an integral $\int f'(x)g'(x)dx$ that we can't evaluate to an integral $\int f(x)g(x)dx$ that we can evaluate.

   (d) (2 points) TRUE $\boxed{\text{FALSE}}$ When finding the area between the curves $y = x^3$ and $y = x^2$ it suffices to find the value of the definite integral $\int_{-1}^{1} \left[x^3 - x^2\right]\, dx$, and then take the absolute value of this value to get the right answer.

   (e) (2 points) $\boxed{\text{TRUE}}$ FALSE If f is a continuous, increasing function, then the left-hand Riemann sum method always underestimates the definite integral.

2. Evaluate each indefinite integral.

   (a) (15 points) $\int \sin^3(5x) \cos^2(5x)\, dx$

$$\boxed{\begin{aligned} u &= \cos(5x) \\ du &= -5\sin(5x)\,dx \end{aligned}}$$

$$= \int \sin^2(5x)\cos^2(5x)\cdot\sin(5x)\,dx$$

$$= \int (1-\cos^2(5x))\cos^2(5x)\cdot\sin(5x)\,dx$$

$$= -\frac{1}{5}\int (1-u^2)\cdot u^2\,du = -\frac{1}{5}\int u^2-u^4\,du = -\frac{1}{5}\left(\frac{1}{3}u^3-\frac{1}{5}u^5\right)+C$$

$$= \boxed{-\frac{1}{15}\cos^3(5x)+\frac{1}{25}\cos^5(5x)+C}$$

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(b) (15 points) $\int \frac{\cos(\ln x)}{x}\,dx$

u-sub

$$\boxed{\begin{aligned} u &= \ln x \\ du &= \frac{1}{x}\,dx \end{aligned}}$$

$$\begin{aligned} &= \int \cos u\,du \\ &= \sin u + C \\ &= \boxed{\sin(\ln x) + C} \end{aligned}$$

(c) (15 points) $\int x^7 \ln(x)\,dx$

IBP

$$\boxed{\begin{aligned} u &= \ln x & dv &= x^7\,dx \\ du &= \frac{1}{x}\,dx & v &= \frac{1}{8}x^8 \end{aligned}}$$

$$\begin{aligned} &= \frac{x^8}{8}\cdot \ln x - \int \frac{1}{8}x^8 \cdot \frac{1}{x}\,dx \\ &= \frac{x^8}{8}\ln x - \frac{1}{8}\int x^7\,dx \\ &= \boxed{\frac{x^8}{8}\ln x - \frac{1}{64}x^8 + C} \end{aligned}$$

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3. (15 points) Using the general Riemann Sum method, $A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$, evaluate the definite integral below. You may assume there are n equally spaced subintervals. Choose x_i^* to be the right-hand endpoint of each interval. ANY OTHER METHOD WILL RECEIVE NO CREDIT!

$$\int_0^2 (3x^2 + 5)\, dx$$

$$R_n = \sum_{i=1}^{n} f(x_i) * \Delta x \qquad \Delta x = \frac{2}{n} \qquad x_i = \frac{2i}{n}$$

$$= \sum_{i=1}^{n} f\left(\frac{2i}{n}\right) \cdot \frac{2}{n}$$

$$= \frac{2}{n} \sum_{i=1}^{n} \left(3\left(\frac{2i}{n}\right)^2 + 5\right) = \frac{24}{n^3} \sum_{i=1}^{n} i^2 + \frac{10}{n} \sum_{i=1}^{n} 1$$

$$= \frac{\cancel{24}^{4}}{n^3} \cdot \frac{n(n+1)(2n+1)}{\cancel{6}} + \frac{10}{\cancel{n}} \cdot \cancel{n} = \frac{4n(n+1)(2n+1)}{n^3} + 10$$

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \frac{4n(n+1)(2n+1)}{n^3} + 10 = 8 + 10 = \boxed{18}$$

Check w/ FTC

$$\int_0^2 3x^2 + 5\, dx = x^3 + 5x \Big|_0^2 = (8+10) - 0 = \boxed{18}$$ ✓

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4. (15 points) Find $F'(0)$ for the function below.

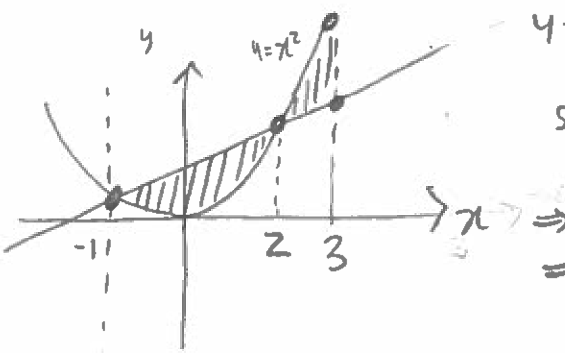
$$F(x) = \int_{x^3}^{\tan x} \left(\frac{1}{1+\sqrt{t}} \right) dt.$$

$$F'(x) = \frac{1}{1+\sqrt{\tan x}} * \sec^2 x - \frac{1}{1+\sqrt{x^3}} * 2x^2$$

$$F'(0) = \frac{1}{1+\sqrt{0}} * 1^2 - \cancel{\frac{1}{1+0} * 2 \cdot 0} \; 0$$

$$= \boxed{1}$$

5. (15 points) Find the total area bounded by the curves $y = x^2$ and $y = x + 2$ and the lines $x = -1$ and $x = 3$.



$y = x+2$

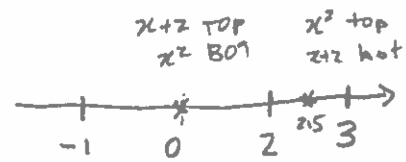
Set $\underline{y=y}$

$x + 2 = x^2$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x - 2)(x+1) = 0$

$\Rightarrow x = 2, -1$



$$\text{Area} = \int_{-1}^{2} (x+2-x^2)\,dx + \int_2^3 (x^2 - x - 2)\,dx$$

$$= \tfrac{1}{2}x^2 + 2x - \tfrac{1}{3}x^3 \Big|_{-1}^{2} + \tfrac{1}{3}x^3 - \tfrac{1}{2}x^2 - 2x \Big|_2^3$$

$$= (2+4 - 8/3) - (\tfrac{1}{2} - 2 + \tfrac{1}{3}) + (9 - \tfrac{9}{2} - 6) - (\tfrac{8}{3} - 2 - 4)$$

$$= 17 - \tfrac{16}{3} - \tfrac{1}{3} - \tfrac{1}{2} - \tfrac{9}{2} = 12 - \tfrac{17}{3} = \tfrac{33-17}{3} = \boxed{19/3}$$