VERSION B Name: KEY

1. (15 points) Using the general Riemann Sum method, $A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$, evaluate the definite integral below. You may assume there are n equally spaced subintervals. Choose x_i^* to be the right-hand endpoint of each interval. ANY OTHER METHOD WILL RECEIVE NO CREDIT!

$$\int_0^3 (3x^2 + 1)\, dx$$

$$R_n = \sum_{i=1}^{n} f(x_i) * \Delta x \qquad \Delta x = \frac{b-a}{n} = \frac{3}{n}$$

$$x_i = \frac{3i}{n}$$

$$= \sum_{i=1}^{n} \left(3\left(\frac{3i}{n}\right)^2 + 1\right) * \frac{3}{n}$$

$$= \frac{3}{n} \cdot \sum_{i=1}^{n} \frac{27 i^2}{n^2} + \frac{3}{n} \cdot \sum_{i=1}^{n} 1.$$

$$R_n = \frac{81}{n^{3/2}} \cdot \frac{\cancel{n}(n+1)(2n+1)}{6} + \frac{3}{\cancel{n}} \cdot \cancel{n} = \frac{27}{2} \frac{(n+1)(2n+1)}{n^2} + 3$$

$$\int_0^3 (3x^2+1)\, dx = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \frac{27}{2} \frac{(n+1)(2n+1)}{n^2} + 3 = \frac{27}{2} * 2 + 3$$

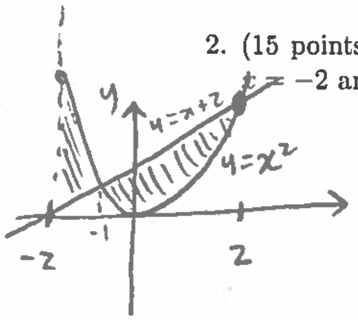
$$= 27 + 3 = \boxed{30} \checkmark$$

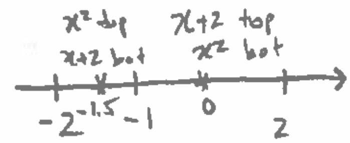
Check w/ FTC

$$\int_0^3 (3x^2+1)\, dx = x^3 + x \Big|_0^3 = (27+3) - 0 = \boxed{30} \checkmark$$

VERSION B Name: ______________________________

2. (15 points) Find the total area bounded by the curves $y = x^2$ and $y = x + 2$ and the lines $x = -2$ and $x = 2$.





Set $y=y$

$$x^2 = x+2$$

$$\Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow (x-2)(x+1) = 0$$

$$\Rightarrow x = 2, -1$$

$$\text{Area} = \int_{-2}^{-1} (x^2 - x - 2) + \int_{-1}^{2} x + 2 - x^2 \, dx = \tfrac{1}{3}x^3 - \tfrac{1}{2}x^2 - 2x \Big|_{-2}^{-1} + \tfrac{1}{2}x^2 + 2x - \tfrac{1}{3}x^3 \Big|_{-1}^{2}$$

$$= \left(-\tfrac{1}{3} - \tfrac{1}{2} + \cancel{2}\right) - \left(-\tfrac{8}{3} - \cancel{2} + 4\right) + \left(2 + 4 - \tfrac{8}{3}\right) - \left(\tfrac{1}{2} - 2 + \tfrac{1}{3}\right)$$

$$= 8 - 1 - 2/3 = 6 + 1/3 = \boxed{19/3}$$

3. **True or False?** Determine if each statement below is always true or sometimes false. Circle your answer to each statement.

(a) (2 points) TRUE $\boxed{\text{FALSE}}$ When finding the area between the curves $y = x^3$ and $y = x^2$ it suffices to find the value of the definite integral $\int_{-1}^{1} [x^3 - x^2] \, dx$, and then take the absolute value of this value to get the right answer.

(b) (2 points) $\boxed{\text{TRUE}}$ FALSE If f is a continuous function, then the function $F(x) = \int_a^x f(t)dt$ is an anti-derivative of f.

(c) (2 points) TRUE $\boxed{\text{FALSE}}$ If f is a continuous, increasing function, then the right-hand Riemann sum method always underestimates the definite integral.

(d) (2 points) TRUE $\boxed{\text{FALSE}}$ To evaluate $\int x \tan^{-1}(x) \, dx$ by parts, choose $u = x$ and $dv = \tan^{-1}(x) \, dx$.

(e) (2 points) TRUE $\boxed{\text{FALSE}}$ The goal of integration by parts is to go from an integral $\int f'(x)g'(x)dx$ that we can't evaluate to an integral $\int f(x)g(x)dx$ that we can evaluate.

VERSION B

Name: ______________________

4. Evaluate each indefinite integral.

IBP

(a) (15 points) $\int x^5 \ln(x)\, dx$

$$\boxed{\begin{array}{ll} u = \ln x & dv = x^5\, dx \\ du = \frac{1}{x}\, dx & v = \frac{1}{6}x^6 \end{array}}$$

$$= \frac{1}{6}x^6 \cdot \ln x - \int \frac{1}{6} x^{\cancel{6}\,5} \cdot \frac{1}{\cancel{x}}\, dx$$

$$= \boxed{\frac{1}{6}x^6 \ln x - \frac{1}{36}x^6 + C}$$

(b) (15 points) $\int \sin^3(8x)\cos^2(8x)\, dx$ $= \int \sin^2(8x)\cos^2(8x) \cdot \sin(8x)\, dx$

$$= \frac{-1}{8}\int (1-\cos^2(8x)) \cdot \cos^2(8x) \cdot (-8\sin(8x))\, dx$$

$$\boxed{\begin{array}{l} u = \cos 8x \\ du = -8\sin 8x\, dx \end{array}}$$

$$= -\frac{1}{8}\int (1-u^2)u^2\, du = -\frac{1}{8}\int u^2 - u^4\, du$$

$$= \frac{-1}{8}\cdot\frac{1}{3}u^3 - \frac{-1}{8}\cdot\frac{1}{5}u^5 + C$$

$$= \boxed{-\frac{1}{24}\cos^3(8x) + \frac{1}{40}\cos^5(8x) + C}$$

VERSION B Name: ______________________

(c) (15 points) $\int \frac{\cos(\ln x)}{x}\, dx$

$$\boxed{\begin{gathered} u = \ln x \\ du = \frac{1}{x}\, dx \end{gathered}}$$

$$= \int \cos(u)\, du$$

$$= \sin(u) + C$$

$$= \boxed{\sin(\ln x) + C}$$

5. (15 points) Find $F'(0)$ for the function below.

$$F(x) = \int_{\tan x}^{x^2} \left(\frac{1}{1+\sqrt{t}} \right) dt.$$

$$F'(x) = \frac{1}{1+\sqrt{x^2}} \cdot 2x - \frac{1}{1+\sqrt{\tan x}} \cdot \sec^2 x$$

$$F'(0) = \cancel{\frac{1}{1+0} \cdot 2 \cdot 0} \to 0 - \frac{1}{1+0} \cdot 1^2$$

$$= \boxed{-1}$$