Quiz 9

1. Suppose 40% of women in a community work currently. Next generation, 80% of the daughters of working women will themselves become working women, and 20% of the daughters of non-working women will become working women. What percentage of women will NOT work in the next generation? What percentage will work next generation? (7 pts.)

transition matrix * initial distribution = generation 1 distribution

$$\begin{matrix} & W & NW \\ W & \\ NW & \end{matrix}\begin{bmatrix} .8 & .2 \\ .2 & .8 \end{bmatrix}\begin{bmatrix} .4 \\ .6 \end{bmatrix} = \begin{bmatrix} .44 \\ .56 \end{bmatrix}$$

next generation
- 56% will NOT work
- 44% will work

2. Find the *stable distribution* X such that $AX = X$ where $A = \begin{bmatrix} .7 & .1 \\ .3 & .9 \end{bmatrix}$. (8 pts.)

want $X = \begin{bmatrix} x \\ y \end{bmatrix}$ such that $AX = X$ and $x + y = 1$

So solve (idea) (work)

$$.7x + .1y = x$$
$$.3x + .9y = y$$
$$x + y = 1$$

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ -.3 & .1 & 0 \\ .3 & -.1 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 1 \\ -3 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

ANS
$X = \begin{pmatrix} 1/4 \\ 3/4 \end{pmatrix}$
or
$x = 1/4 \quad y = 3/4$

$(A - I)X = 0$

$$\begin{bmatrix} -.3 & .1 \\ .3 & -.1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

and $\begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$

$$\sim \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 4 & 3 \\ 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & 3/4 \\ 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 1/4 \\ 0 & 1 & 3/4 \\ 0 & 0 & 0 \end{array}\right]$$