Math 1552, Integral Calculus Section 8.8: Improper integrals, and Section 4.5: L'Hopital's Rule

1. Determine which of the following improper integrals converge and which diverge.

(a)
$$\int_4^{1} \frac{3}{\sqrt{x-2}} dx$$

(b)
$$\int_{-3}^{4} \frac{5}{(x+2)^2} dx$$

2. Explain in words why each of the integrals in parts (*a*)-(*b*) converge and why some diverge. What would the answers be if the square root of (*a*) and the squaring of (*b*) were reversed?

Evaluate the following limits using L'Hopital's Rule.

 $3. \lim_{x\to 0} \left[\frac{1}{x} - \cot x \right]$

4. $\lim_{x\to 0^+} [x(\ln(x))^2]$

5. $\lim_{x \to \infty} (x + e^x)^{2/x}$

ANSWERS:

- 1a. $+\infty$ dne
- 1b. $+\infty$ dne
- 2. In (*a*) the denominator increases to infinity slowly, so the fraction decreases to zero slowly, so the net effect is for the improper integral to diverge since the area "adds up" to infinity. In part (*b*), the denominator goes to zero quickly near x = -2, so the net effect is for the fraction to blow up to infinity quickly, which also results in a divergent improper integral. If the problems would have had the roles reversed with the square root in (*b*) and the squared in (*a*), then the improper integrals would each evaluate to a finite area in these cases so the answer would be CONVERGE in both cases (with the $\sqrt{}$ and ()² reversed).
- 3.0
- 4. 0
- 5. *e*²