January 26, 2000

reading the dual solution from the final tableau or dictionary

Executing the simplex algorithm on our example, we end with the following optimal basic feasible solution:

<table>
<thead>
<tr>
<th>column:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>BASIS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-5</td>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>-2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>-1</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>-21</td>
</tr>
</tbody>
</table>

The optimal dual solution (0,1,1,0) is read from the last row.

Why this works

click on link "Final tableau solves the dual" for a detailed explanation.

Lagrange multiplier interpretation of dual solution

Example: Consider again

(P) max $3x_1 + 2x_2$ subject to $x_1 \geq 0$ and $x_2 \geq 0$

$-x_1 + 2x_2 \leq 4$

$x_1 + x_2 \leq 8$

$2x_1 + x_2 \leq 13$

$x_1 - x_2 \leq 5$

We saw that (5,3) and (0,1,1,0) were solutions for (P) and (D). Draw picture.

Example: Suppose with the same constraints, we use the objective function $3x_1 + 4x_2$. Executing the simplex algorithm, we end up with the following final tableau:

<table>
<thead>
<tr>
<th>column:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>BASIS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>1/3</td>
<td>-5/3</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>2/3</td>
<td>-1/3</td>
<td>0</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>1/3</td>
<td>1/3</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0</td>
<td>-1/3</td>
<td>2/3</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>-1/3</td>
<td>-10/3</td>
<td>0</td>
<td>0</td>
<td>-28</td>
</tr>
</tbody>
</table>

notice that $(3,4) - (1/3)(-1,2) - 10/3(1,1) = (0,0)$.
Example: Suppose with the same constraints, we use the objective function 
\(-3x_1 + 2x_2\). Executing the simplex algorithm, we end up with the following 
final tableau:

<table>
<thead>
<tr>
<th>column:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>BASIS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-1/2</td>
<td>1</td>
<td>1/2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>3/2</td>
<td>0</td>
<td>-1/2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>5/2</td>
<td>0</td>
<td>-1/2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>1/2</td>
<td>0</td>
<td>1/2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>-2</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-4</td>
</tr>
</tbody>
</table>

notice that \((-3,2) - (1)(-1,2) = (-2,0)\) is "normal" to the set \(x \geq 0\).

General duality scheme
follow this link

Example:

primal: 
dual:

max\n\[3x_1 - 5x_2 + 7x_3 \text{ (objective)}\]
\[x_1 + x_2 - 3x_3 \leq 11\]
\[2x_1 + 8x_2 + 5x_3 \leq 91\]
\[x_1 + 2x_2 + 3x_3 = 15\]
\[x_1 - x_2 + 7x_3 = 9\]
\[4x_1 + x_2 - 12x_3 = 5\]
\[x_1 \geq 0, x_2, x_3 \text{ unrestricted}\]

min\n\[11y_1 + 91y_2 + 15y_3 + 9y_4 + 5y_5 \text{ (objective)}\]
\[y_1 + 2y_2 + y_3 + y_4 + 4y_5 \geq 3\]
\[y_1 + 8y_2 + 2y_3 - y_4 + y_5 = -5\]
\[-3y_1 + 5y_2 + 3y_3 + 7y_4 - 12y_5 \geq 7\]
\[y_1, y_2 \geq 0, y_3, y_4, y_5 \text{ unrestricted}\]

HW#15: Find an LP of the form
\((P)\) max \(c_1 x_1 + c_2 x_2\) subject to \(x_1 \geq 0\) and \(x_2 \geq 0\)
\[-x_1 + 2x_2 \leq 4\]
\[x_1 + x_2 \leq 8\]
\[2x_1 + x_2 \leq 13\]
\[x_1 - x_2 \leq 5\]
such that \(y=0\) solves the dual problem.

HW#16: For the linear programming problem

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(P) maximize $3x_1 + 2x_2 + x_3$
subject to the constraints
$x_1 + 2x_2 + x_3 \leq 5$
$2x_1 - x_2 + 4x_3 \leq 2$
$x_1 + x_2 + x_3 \leq 3$
$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

explain why $(5/3, 4/3, 0)$ and $(0, 1/3, 7/3)$ solve (P) and its dual, respectively. Do not actually solve (P) or its dual. Just present an absolutely convincing argument that these are solutions.

HW#17: Make up a linear programming problem in the variables $x_1$ and $x_2$ in standard form, for which the solution occurs at $(5, 2)$, and the dual solution is $(0, 3, 5, 0)$. 

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