Math. 4581, Homework assignment, due on November 19, 2018.

1.(5 pts) Find the Fourier series for the function

$$f(x) = \begin{cases} 2x & -\dfrac{\pi}{2} < x < \dfrac{\pi}{2} \\ 0 & -\pi < x < -\dfrac{\pi}{2}, \dfrac{\pi}{2} < x < \pi. \end{cases}$$

2.(5 pts) Solve the boundary value problem

$$\begin{cases} u_{xx} + u_{yy} = 0 \quad \text{for } 0 < x, y < 1 \\ u_x(0, y) = 0, \;\; u(1, y) = 0 \quad \text{for } 0 < y < 1 \\ u(x, 0) = 1, \;\; u_y(x, 1) = 0 \quad \text{for } 0 < x < 1. \end{cases}$$

3.(5 pts) Find the bounded solution of Laplace's equation in the region $\Omega = \{(r, \theta) : r > 1, 0 < \theta < \pi\}$ subject to the boundary conditions $u(r, \pi) = u(r, 0) = 0$ for $r > 1$ and $u(1, \theta) = 1$ for $0 < \theta < \pi$.

4.(5 pts) Let $D > 0$. Find u satisfying

$$\begin{cases} u_t(t, x) = Du_{xx}(t, x) \quad \text{for } 0 < x < 1, t > 0 \\ u(0, x) = \begin{cases} 2x & 0 < x < \dfrac{1}{2} \\ 1 & \dfrac{1}{2} < x < 1, \end{cases} \\ u(t, 0) = 0, \;\; u_x(t, 1) = 0 \quad \text{for } t > 0. \end{cases}$$

5.(5 pts) Solve the initial boundary value problem

$$\begin{cases} u_t = Du_{xx} + \alpha u \quad 0 < x < 1, t > 0 \\ u(0, x) = x(1 - x) \quad 0 < x < 1 \\ u(t, 0) = 0, \quad u_x(t, 1) = 0 \quad t > 0, \end{cases}$$

where $D, \alpha > 0$.

6.(5 pts) Let u satisfy

$$\begin{cases} u_{tt} = a^2 u_{xx} \quad 0 < x < 1, t > 0 \\ u(0, x) = x^2(1 - x)^2, \;\; u_t(0, x) = 0 \quad 0 < x < 1 \\ u_x(t, 0) = 0, \quad u_x(t, 1) = 0 \quad t > 0. \end{cases}$$

Evaluate $u(t, 0.3)$ at $t = 5/a, t = 7/a$, and $t = 10/a$.

7.(5 pts) Solve the initial boundary value problem

$$\begin{cases} u_{tt} = 4u_{xx} \quad x > 0, t > 0 \\ u(x,0) = \dfrac{x^2}{8}, \; u_t(x,0) = x \quad x \geq 0 \\ u(0,t) = t^2 \quad t \geq 0. \end{cases}$$

8.(5 pts) Use the parallelogram rule to evaluate $u(1, 0.3)$ and $u(1.3, 0.3)$ if u satisfies

$$\begin{cases} u_{tt} = 4u_{xx} \quad 0 < x < 1, t > 0 \\ u(0,x) = x(x-1), \; u_t(0,x) = 0 \quad 0 < x < 1 \\ u(t,0) = 4t^2, \quad u(t,1) = 0 \quad t > 0. \end{cases}$$

9.(5 pts) Use Laplace transforms to solve the boundary value problem

$$\begin{cases} Y_{xx}(t,x) - 2Y_{tx}(t,x) + Y_{tt}(t,x) = 0, \quad 0 < x < 1, \, t > 0, \\ Y(0,x) = Y_t(0,x) = 0, \quad 0 < x < 1, \\ Y(t,0) = 0, Y(t,1) = F(t), \quad t > 0. \end{cases}$$