

Math. 4581, Suggested homework problems 2.

1. Find a (double) generalized Fourier series in $L^2((0, \pi) \times (0, \pi))$ in terms of the orthogonal approximating basis $\{\sin mx \cos ny : m = 1, 2, \dots, n = 0, 1, 2, \dots\}$ for the following functions:

(a)

$$f(x, y) = \begin{cases} 1 & \text{for } 0 < x < \pi, 0 < y < \frac{\pi}{2} \\ 0 & \text{for } 0 < x < \pi, \frac{\pi}{2} < y < \pi. \end{cases}$$

(b)

$$g(x, y) = xy.$$

Answers:

(a)

$$f(x, y) = \sum_{m=1}^{\infty} \frac{1 - (-1)^m}{\pi m} \sin mx + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{4[1 - (-1)^m] \sin(\frac{n\pi}{2})}{\pi^2 mn} \sin mx \cos ny.$$

(b)

$$g(x, y) = \sum_{m=1}^{\infty} \frac{\pi(-1)^{m+1}}{m} \sin mx + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{4[1 - (-1)^n](-1)^m}{\pi mn^2} \sin mx \cos ny.$$