

① Use D'Alembert's formula to find $u(\frac{1}{2}, 2)$ and $u(1, 3.4)$

$$\text{if } \begin{cases} u_{tt} - 4u_{xx} = 0 & 0 < x < 2, t > 0 \\ u(x, 0) = x, u_t(x, 0) = x & 0 < x < 2 \\ u(0, t) = u(2, t) = 0 & t > 0 \end{cases}$$

Notice: Solution is not smooth here and it is in fact discontinuous along some characteristic lines.

② Use the parallelogram rule to find $u(\frac{\pi}{2}, \pi)$ and $u(\frac{\pi}{4}, \frac{\pi}{2})$ if

$$\begin{cases} u_{tt} - u_{xx} = 0 & 0 < x < \pi, t > 0 \\ u(x, 0) = \sin x, u_t(x, 0) = 0 & 0 < x < \pi \\ u(0, t) = t^3, u(\pi, t) = t^4 & t > 0 \end{cases}$$

Answers: ① $\frac{1}{2}, -1.4$

$$\text{② } u\left(\frac{\pi}{2}, \pi\right) = \frac{\pi^3}{8} + \frac{\pi^4}{16} - 1, \quad u\left(\frac{\pi}{4}, \frac{\pi}{2}\right) = \frac{\pi^3}{64}$$