Math. 4581, Practice Test 1

1. Let

$$f(x) = \begin{cases} 1 - x & \text{for } 0 < x < 1 \\ -1 - x & \text{for } -1 < x < 0. \end{cases}$$

(a) Sketch the graph on the interval $(-2, 2)$ of the periodic extension $\tilde{f}(x)$ of $f(x)$ and the graph of $[\tilde{f}(x)]'$.
(b) Calculate the Fourier coefficients a_n and b_n for $f(x)$.
(c) Does the differentiated Fourier series of $f(x)$ converge to $[\tilde{f}(x)]'$?
2. Is $\{1, \sqrt{2}\sin \pi x, ..., \sqrt{2}\sin 5\pi x\}$ an orthonormal family in $L^2(0, 1)$? Does $f(x) = (1 + \cos \pi x)\sin \pi x$ belong to the space spanned by the family?
3.(a) Compute all the eigenvalues and the corresponding eigenfunctions for the Sturm-Liouville problem

$$u''(x) + \lambda u(x) = 0, \quad u'(0) = u'(\frac{\pi}{2}) = 0.$$

(b) Let u_1 and u_2 be the eigenfunctions corresponding to the two smallest eigenvalues. Find the projection (in $L^2(0, \frac{\pi}{2})$) of $f(x) = \sin 2x$ onto the subspace spanned by u_1 and u_2.