Math. 4581, Practice Test 2.   SOLUTIONS

2. Using separation of variables we obtain

$$u(t,x) = \sum_{n=1}^{\infty} A_n e^{-9(n\pi)^2 t} \sin n\pi x.$$

Now

$$A_n = 2\left[\int_0^{\frac{1}{2}} x \sin n\pi x \, dx + \int_{\frac{1}{2}}^{1} (1-x) \sin n\pi x \, dx\right]$$

$$= 2\left[-x\frac{\cos n\pi x}{n\pi}\bigg|_0^{\frac{1}{2}} + \int_0^{\frac{1}{2}} \frac{\cos n\pi x}{n\pi} dx - (1-x)\frac{\cos n\pi x}{n\pi}\bigg|_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} \frac{\cos n\pi x}{n\pi} dx\right]$$

$$= -\frac{\cos\frac{n\pi}{2}}{n\pi} + 2\frac{\sin\frac{n\pi}{2}}{(n\pi)^2} + \frac{\cos\frac{n\pi}{2}}{n\pi} + 2\frac{\sin\frac{n\pi}{2}}{(n\pi)^2} = 4\frac{\sin\frac{n\pi}{2}}{(n\pi)^2}.$$

Therefore if $n := 2n$, $A_{2n} = 0$ and if $n := 2n-1$,

$$A_{2n-1} = \frac{4(-1)^{n+1}}{\pi^2(2n-1)^2}$$

and so we can write the solution as

$$u(t,x) = \sum_{n=1}^{\infty} \frac{4(-1)^{n+1}}{\pi^2(2n-1)^2} e^{-9((2n-1)\pi)^2 t} \sin(2n-1)\pi x.$$