1 0.1

1) Consider the following atomic statements:

\[ A = \text{“it looks like a duck”} \]
\[ B = \text{“it swims like a duck”} \]
\[ C = \text{“it quacks like a duck”} \]
\[ D = \text{“it is a duck.”} \]

a) Write down the contrapositive of “if it looks like a duck, swims like a duck, and quacks like a duck, then it is a duck” in ordinary language and in symbolic notation (ie. with ands, ors, nots, if/thens, etc.) Can you write it using only ors and nots?

b) Repeat for the converse of the statement in part a).

2) Write down (in ordinary language) the negation of the statement “for every sports team in New England, there exists a sports team in Atlanta that is better.”

3) Is it possible for an implication and its converse to both be false? (Hint: write out the truth tables for \( p \Rightarrow q \) and \( q \Rightarrow p \).

2 0.2

1) Without using induction, show that \( n^2 - 5n - 14 \) is an even integer whenever \( n \) is a natural number.

2) Let \( a \) and \( b \) be integers. Find necessary and sufficient conditions for \( a^2 - b^2 \) to be odd (hint: consider cases.)

3) If \( a \) is a rational number and \( b \) is an irrational number, is \( a + b \) necessarily an irrational number?
3 1.1

1) Find the truth value of the compound proposition

\[(p \land q) \Rightarrow r \Rightarrow (\neg r \lor (q \Rightarrow (p \land r)))\]

when \(p\) is true, \(q\) is true, and \(r\) is true.

2) Explain why \(p \Rightarrow p\) is a tautology. Is \(p \Rightarrow \neg p\) a contradiction? What about \(\neg p \Rightarrow p\)? What about \(\neg p \Rightarrow \neg p\)?

4 1.2

1) For each pair of propositions, answer true/false as to whether they are equivalent. If they are not, supply the corresponding rows of their truth tables at which their truth values differ:

\[
\begin{align*}
\text{a) } & \neg
\text{b) } & \neg(p \Rightarrow q) \land \neg(p \lor q) \\
\text{c) } & \neg(p \Rightarrow q) \land (p \lor q) \\
\text{d) } & \neg(p \Rightarrow q) \land (p \land q) \\
\text{e) } & (p \Rightarrow q) \Rightarrow r \land (p \Rightarrow (q \Rightarrow r))
\end{align*}
\]

5 1.3

1) Is the following argument valid?

If I like mathematics, then I will study.

Either I don’t study or I pass mathematics.

If I don’t graduate, then I didn’t pass mathematics.

If I like mathematics, then I will graduate.

6 Challenge Question

1) Consider the operation \(|\) defined by the following truth table:

\[
\begin{array}{c|c|c}
p & q & p | q \\ 
T & T & F \\ 
T & F & T \\ 
F & T & T \\ 
F & F & T \\ 
\end{array}
\]

In other words, \(p | q = \neg(p \land q)\). Write an equivalent expression for \(\neg p\) that only uses the symbols \(|\), \(p\), and parentheses. Do the same for \(\lor\) and \(\land\).