1) Prove the following equation is true for all \( n \in \mathbb{N} \):
\[
(1 + 2 + 3 + \cdots + n)^2 = 1^3 + 2^3 + \cdots + n^3
\]

2) Prove that any \( 2^n \times 2^n \) grid may be tiled by removing a \( 1 \times 1 \) square and covering the remaining squares with non-overlapping “L” shapes of length and height 2. An example with \( n = 2 \) is depicted below.

3) Debug the following “proof” of the claim that all horses are the same color:
   - **Base case** \( n = 1 \) there is only one horse, so clearly all \( n \) horses have the same color
   - **Inductive step** Assume the claim is true for any collection of \( n \) horses. Consider some collection of \( n + 1 \) horses.
     
     By inductive hypothesis, the first \( n \) horses are all the same color. Also by inductive hypothesis, the last \( n \) horses are the same color, so all \( (n + 1) \) horses are the same color.

4) Consider the Fibonacci sequence 0, 1, 2, 3, 5, 8, 13, \ldots. Formally, this sequence is defined by a linear recurrence with seed values \( f_0 = 0, f_1 = 1 \):
\[
f_n = f_{n-1} + f_{n-2} \quad \text{for } n \geq 2.
\]
Prove that
\[
f_n = \frac{(1 + \sqrt{5})^n + (1 - \sqrt{5})^n}{\sqrt{5}}
\]
for all \( n \in \{0\} \cup \mathbb{N} \).