Problem 1. (10pts) (a) Compute the gcd(2628, 738) using Euclid’s gcd algorithm.

(b) Show that if $a^3 | b^2$ then $a | b$, where $a$ and $b$ are positive integers.
Problem 2. (10 pts) Let $d = \gcd(m, n)$, where $m$ and $n$ are positive integers.

(a) Prove or disprove: Suppose $c$ is a common divisor of $m$ and $n$. Then $c$ divides $d$.

(b) We saw in class that $d$ can be written as $mx + ny$ where $x$ and $y$ are integers. Can a positive integer smaller than $d$ be written as $mx_1 + ny_1$ for some integers $x_1$ and $y_1$? (Recall that $d$ divides both $m$ and $n$). Explain your answer.
Problem 3. (10 pts) Let $\{a_n\}$, for $n = 0, 1, 2, \ldots$ be a sequence satisfying $a_n = 2a_{n-1} + 3a_{n-2}$, for $n \geq 3$. Given that $a_1 = a_2 = 1$, prove that $a_n = \frac{1}{5}(3^{n-1} - (-1)^n)$ for $n \geq 1$. 
Extra Credit. (5 pts) The Fibonacci sequence \( \{F_n\} \) is defined as follows. \( F_0 = F_1 = 1 \). For \( n \geq 2, F_n \), the \( n \)th Fibonacci number satisfies the recurrence:

\[
F_n = F_{n-1} + F_{n-2}.
\]

Show that every pair of consecutive Fibonacci numbers is relatively prime. (Namely, show that \( \gcd(F_n, F_{n-1}) = 1 \) for \( n \geq 1 \).)