1. Show that the gcd of $8a + 3$ and $5a + 2$ is equal to 1 for all positive integers $a$.

2. Prove or disprove: if $g = \gcd(m, n)$, then $\gcd(m/g, n/g) = 1$.

3. Define a bijection.

4. Given integers $a, b,$ and $n$, when is there an integer solution to $ax \equiv b \pmod n$?

5. What do we mean by the inverse of an integer $b \pmod n$?

6. Define the Euler $\phi$ function.

7. (a) Compute the Euler $\phi$-function of the following integers: 15, 19, 27.

   (b) For which integers $m, n$, is it the case that $\phi(mn) = \phi(m)\phi(n)$?

8. Suppose that $e = 3$ and $n = 23 \times 47$ in Alice’s RSA cryptosystem.
   Find Alice’s decrypting exponent $d$.

9. Find the smallest nonnegative integer $x$ that satisfies the system of congruences:

   
   \[
   \begin{align*}
   x &\equiv 6 \pmod 8 \\
   x &\equiv 17 \pmod {25}
   \end{align*}
   \]

10. What is the computational significance of the Chinese Remainder Theorem?

11. Is every function from the set of natural numbers to \{0, 1\} computable in a given programming language?

12. Is $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = 5x - 2|x|$ a bijection? (\(\mathbb{R}\) represents the set of reals.)