The following exercises are all from the Grimmett-Stirzaker book (3rd ed., 2001). Please stop by my office, if you don’t have a copy of the book. I can give a copy of the problems.

Problem 1. Exercise 1.4.5

(This is just to test basic knowledge of conditional probability, and since this is a classic problem.)

Problem 2. Exercise 1.8.16

Problem 3. Exercise 1.8.17

Problem 4. Exercise 1.8.18

Problem 5. (a) Let $\Omega$ be a set and let $A_1$ and $A_2$ be subsets of $\Omega$. Show that the smallest $\sigma$-algebra containing $A_1$ and $A_2$ consists of at most 16 sets.

(b) Let $A_1, A_2, \ldots, A_k$ be subsets of $\Omega$. Let $\mathcal{F}_\parallel$ be the smallest $\sigma$-algebra containing the $A_i$’s. Show that $\mathcal{F}_\parallel$ has at most $2^{2^k}$ members.

(c) Show that the upper bound in part (b) can not be improved. (Hint: Let $M$ be the $k$-element set $\{p_1, \ldots, p_k\}$, and let $\Omega = 2^M$ be the set of subsets of $M$. Let $A_i$ be all subsets of $M$ that contain the point $p_i$.)