

HOMEWORK #3
Math 6014

Problem 4. A non-negative real matrix M is *doubly stochastic* if the sum of the entries in each row of M is 1 and the sum of the entries in each column of M is 1. A *permutation matrix* is a 0, 1-matrix which has exactly one 1 in each row and each column. Prove that every doubly stochastic matrix M can be expressed as a convex linear combination of permutation matrices; that is,

$$M = a_1P_1 + a_2P_2 + \cdots + a_kP_k,$$

where each P_i is a permutation matrix, each a_i is a non-negative real and $a_1 + a_2 + \cdots + a_k = 1$.

Problem 5. Let G be a bipartite multigraph, and let Δ be its maximum degree. Prove that G has a matching saturating every vertex of degree Δ .

Instructions: You are only allowed to use your own notes, class handouts and the designated textbook. No collaboration. Clarity of exposition, ease of expression, mathematical elegance and overall physical appearance will all be factors in grading. Please start your work on a new page, and sign and attach this sheet. If submitting a pdf file electronically, include the appropriate statement in the body of the message. This assignment is due before 3:05PM, Wednesday, October 8, 2008.

Format: Please type your solution on one-sided letter size paper in 10pt font or larger. Figures and mathematical formulae may be drawn by hand in black ink. Do not fold pages or bend corners.

NAME (please print):

GTid#:

Please check the box that applies.

The attached paper represents my own work. Since the posting of this homework I have worked on my own, have not consulted with other persons, and have not used sources other than those listed above. I understand that making a false statement is a violation of the Georgia Tech Honor Code.

I have consulted with the following persons and used the following sources:

Signature:

Date: