WEEK 12 PROBLEMS

Math 6014

1. Let S be an infinite set in the plane. Prove that there is an infinite set $A \subseteq S$ such that either A is contained in a line, or no three points of A are collinear.

2. Let $n \geq 1$ be a natural number, and let G_n be the graph with vertex-set all 2-element subsets of $\{1, 2, \ldots, n\}$, where for $a < b < c$ the vertex $\{a, b\}$ is adjacent to $\{b, c\}$, and those are all the edges of G_n. Prove that G_n has no triangle, and that $\chi(G_n) \to \infty$ as $n \to \infty$.

3. Let $g_1, g_2, \ldots, g_k$ be bounded real functions, let f be another real function, and let δ and ϵ be positive constants. Assume that $\max_{1 \leq i \leq k}[g_i(x) - g_i(y)] > \delta$ whenever $f(x) - f(y) > \epsilon$. Prove that f is bounded.

4. Let $s, t \geq 2$ be integers, and let T be a tree on t vertices. Prove that if G is a graph on $(s-1)(t-1)+1$ vertices, then either G has a K_s subgraph, or the complement of G has a subgraph isomorphic to T. Prove that the number $(s-1)(t-1)+1$ is best possible.

5. Prove that in every partition of the edges of K_{3n-1} into two sets, one of the sets contains a matching of size n. Prove that the same does not hold for K_{3n-2}.