1. A graph is outerplanar if it is isomorphic to a plane graph such that every vertex is incident with the unbounded face. Prove that a graph is outerplanar if and only if it has no subgraph isomorphic to a subdivision of $K_4$ or $K_{2,3}$.

2. Prove that a 2-connected plane graph is bipartite if and only if every face is bounded by an even cycle.

3. Characterize graphs $H$ such that for every graph $G$ the following holds: $G$ has a minor isomorphic to $H$ if and only if $G$ has a subgraph isomorphic to a subdivision of $H$.

4. A plane graph is a triangulation if every face is bounded by a cycle of length three. Prove that a loopless plane triangulation $G$ has chromatic number 3 if and only if every vertex of $G$ has even degree.

5. Prove that the faces of a Hamiltonian plane graph can be 4-colored in a such a way that whenever two faces are incident with the same edge they receive different colors.

6. Prove that a graph has a $K_5$ or $K_{3,3}$ subdivision if and only if it has a $K_5$ or $K_{3,3}$ minor.

7. Let us say that a graph is ci-plane if it satisfies the definition of a plane graph with the exception that the sets $A$ are not polygonal arcs, but continuous images of $[0,1]$ instead. Prove that every ci-plane graph is planar.

8. Let us say that a graph is cs-plane if it satisfies the definition of a plane graph with the exception that the sets $A$ are not polygonal arcs, but connected subsets of the plane instead. Prove that every graph is isomorphic to a cs-plane graph. (Recall that a topological space $X$ is connected if it has no nonempty proper subset that is both open and closed.)

*Hint.* The set consisting of the origin and all points with coordinates $(x, \sin(1/x))$ for $x > 0$ is connected.