

WEEK 16 PROBLEMS

Math 6014A

1. Let G be a graph, let T be a spanning tree of G , and for $e \in E(G) - E(T)$ let C_e be the edge-set of the unique cycle in $T + e$. Prove that $\{C_e : e \in E(G) - E(T)\}$ is a basis of the cycle space of G .
2. Let G be a graph, let T be a spanning tree of G , and for $e \in E(T)$ let X_e be the vertex-set of one of the components of $T \setminus e$. Prove that $\{\partial(X_e) : e \in E(T)\}$ is a basis of the cut space of G .
3. What are the dimensions of the cycle space and the cut space of a graph with k components?
4. Let A be the adjacency matrix of a graph G , and let $k \geq 0$ be an integer. Prove that the ij entry of A^k is the number of walks from i to j of length exactly k .
5. Let L be the Laplace matrix of a graph G with vertex-set $\{1, 2, \dots, n\}$, and let $g(\mathbf{x}) = \langle L\mathbf{x}, \mathbf{x} \rangle$. Prove that $g(\mathbf{x}) = \sum_{ij \in E(G)} (x_i - x_j)^2$.