WEEK 4 PROBLEMS
Math 6014A

1. Let G be a graph, let $A \subseteq V(G)$, let $u \in V(G) - A$, and let k be an integer. Prove that either
(i) there are k paths, each between u and some vertex of A with only u in common, or
(ii) there is a set $X \subseteq V(G) - \{u\}$ with $|X| < k$ such that $G \backslash X$ has no path between u and A,
and not both.

2. Let $k \geq 2$ be an integer. Prove that if a graph G is k-connected, and $v_1, v_2, \ldots, v_k$ are vertices of G, then there is a cycle in G that contains all v_i, $1 \leq i \leq k$.
Hint. Use induction to get a cycle through $v_1, v_2, \ldots, v_{k-1}$.

3. Prove that if G is 3-regular, then the vertex-connectivity equals the edge-connectivity.

4. Let $k \geq 2$ be an integer. Prove that every connected k-regular bipartite graph on at least three vertices is 2-connected.

5. Prove that any two edges of a 2-connected graph lie on a common cycle.

6. Let G be a 2-connected graph. Prove that its vertices can be numbered as $v_1, v_2, \ldots, v_n$ in such a way that for all $i = 1, 2, \ldots, n$, both sets $\{v_1, v_2, \ldots, v_i\}$ and $\{v_i, v_{i+1}, \ldots, v_n\}$ induce connected graphs.