

## WEEK 9 PROBLEMS

Math 6014A

1. A graph is *chordal* if it has no induced cycle of length four or more. In other words, every cycle of length at least four has a chord. Prove that if  $G$  is a chordal graph and  $X \subseteq V(G)$  is an inclusion-wise minimal set such that  $G \setminus X$  is disconnected, then  $X$  is a clique.
2. Prove that every chordal graph has a simplicial vertex; that is, a vertex whose neighbors form a clique.
3. Prove that every chordal graph is perfect.
4. Prove that the complement of every chordal graph is perfect.
5. Let  $a_1, a_2, \dots$  be a sequence of non-negative real numbers such that  $a_{r+s} \geq a_r + a_s$  and  $a_n/n$  is bounded. Prove that  $\lim_{n \rightarrow \infty} a_n/n$  exists.