1.3. Lemma. *Let T and S be sets of nodes of a matching-covered graph G such that $\nabla(T)$ and $\nabla(S)$ are tight cuts and $|T \cap S|$ is odd. Then $\nabla(T \cap S)$ and $\nabla(T \cup S)$ are also tight cuts. Furthermore, no edge connects $T-S$ to $S-T$.*

1.5. Theorem. *The result of any two tight cut decomposition procedures of the same graph is the same list of bricks and braces.*

Proof. We use induction on $|V(G)|$. Let us consider any two tight cut decomposition procedures and let $\mathscr{F}$ and $\mathscr{F}'$ be the two corresponding families of tight cuts.

Case 1. $\mathscr{F}$ and $\mathscr{F}'$ have a common member C. Then we can start both $\mathscr{F}$ and $\mathscr{F}'$ with breaking along this cut C. This results in the same two graphs G_1 and G_2 in both cases, and so both decompositions produce the union of the result of a tight cut decomposition of G_1 and one of G_2. Since by the induction hypothesis these lists depend on G_1 and G_2 only, this proves the assertion.

Case 2. There are cuts $C \in \mathscr{F}$ and $C' \in \mathscr{F}'$ which are laminar. Let $\mathscr{F}''$ be any maximal set of mutually laminar non-trivial tight cuts containing C and C'. Then by case 1, the decomposition procedures associated with $\mathscr{F}$ and $\mathscr{F}''$, as well as the procedures associated with $\mathscr{F}'$ and $\mathscr{F}''$, end up with the same lists of bricks and braces. Hence $\mathscr{F}$ and $\mathscr{F}'$ end up with the same lists.

Case 3. There are cuts $C=\nabla(T) \in \mathscr{F}$ and $C'=\nabla(T') \in \mathscr{F}'$ such that $|T \cap T'|$ is odd and at least 3. Then Lemma 1.3 shows that $C''=\nabla(T \cap T')$ is also a tight cut, which is clearly non-trivial. Let $\mathscr{F}''$ be a maximal family of mutually laminar non-trivial tight cuts containing C''. Then C'' is laminar with C, and hence the decompositions associated with $\mathscr{F}$ and $\mathscr{F}''$ result in the same list of bricks and braces, and similarly for $\mathscr{F}'$ and $\mathscr{F}''$. This proves the assertion.

To complete the proof, consider any $C \in \mathscr{F}$ and $C' \in \mathscr{F}'$. If they are laminar, we are done by case 2, so assume that they are not laminar. Choose shores T of C and T' of C' such that $|T \cap T'|$ is odd (this is clearly possible). If $T \cap T'$ is not a singleton, then we are done by case 3, so assume that $T \cap T' = \{u\}$ for some node u. If $V(G)-T-T'$ is not a singleton, then again we can apply case 3 by replacing T and T' by their complements. So assume that $V(G)-T-T'=\{v\}$ for some node v.

Now the pair $\{u, v\}$ separates the graph; more exactly, no edge connects $T-T'$ to $T'-T$, by Lemma 1.3. Hence C and C' are two 2-separation cuts defined by the same separating pair of nodes. If $\mathscr{F}=\{C\}$ and $\mathscr{F}'=\{C'\}$ then it is obvious that breaking G along C and along C' results in isomorphic graphs (after deleting multiple edges), and we are done. So assume that, e.g., $\mathscr{F}$ contains another cut $C_0=\nabla(S)$. Then C_0 is laminar with C, and so we may assume without loss of generality that $S \cap T = \varnothing$. If C_0 is laminar with C' then we are again finished by case 2, so assume that this does not happen. Then S is not a subset of T' and hence we must have $v \in S$ and so $|S \cap (V(G)-T')|=1$. Hence trivially $|(V(G)-S) \cap T'|$ is odd. But then we are finished by case 3 since $|(V(G)-S) \cap T'| = |T'-S| \neq 1$, as $C_0 \neq C$. ∎