Forbidden structures for efficient First-Fit chain partitioning

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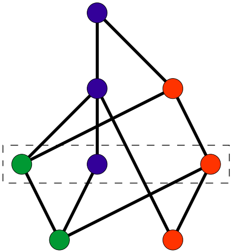
chain partitioning

- $P = (X, \leqslant)$
- antichain, $\mathrm{w}(P)$
- chain
- a chain partition of $P = (X, \leqslant)$ is a family of disjoint chains $C_1, \ldots, C_k$ so that $C_1 \cup \ldots \cup C_k = X$

Theorem (Dilworth 1950)

Every poset P can be partitioned into $\mathrm{w}(P)$ chains.

Example



on-line chain partitioning

- two players: Spoiler and Algorithm,
- the game is played in rounds,

  during each round:
  - Spoiler introduces a new point with his comparability status to already presented points,
  - Algorithm assigns this new point to a chain.

- Algorithm tries to use as small number of colors as possible,
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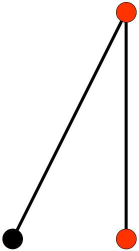

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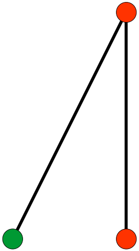

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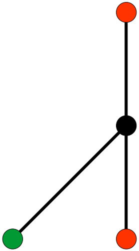

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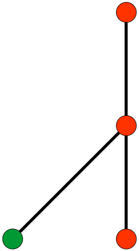

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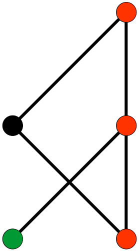

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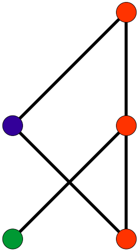

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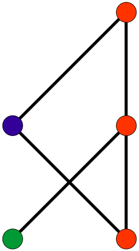

- on-line result = 3 colors

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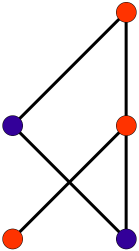

- on-line result = 3 colors
- off-line result = 2 colors

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Estimate:

ocp(w) – the minimum n such that Algorithm has a strategy to partition any poset of width w into at most n chains

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Theorem (Szemerédi 1981; Kierstead 1981)

$$\binom{w+1}{2} \leqslant \mathrm{ocp}(w) \leqslant \frac{5^w - 1}{4}$$

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$$\mathrm{ocp}(w) \leqslant \mathit{poly}(w)?$$

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first-fit – on-line algorithm that always uses the lowest possible number

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Theorem (Kierstead 1986)

There exists a strategy for Spoiler that forces First-Fit to use infinitely many colors even if the game is played on orders of width 2.

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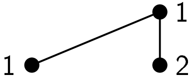

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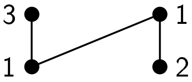

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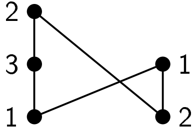

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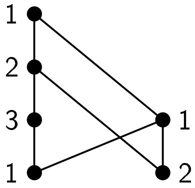

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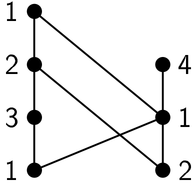

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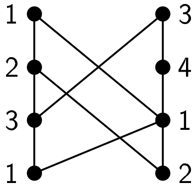

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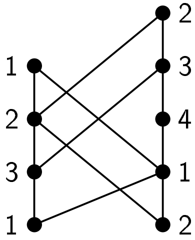

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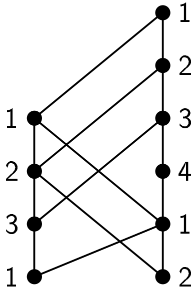

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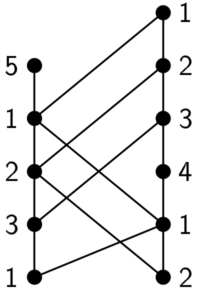

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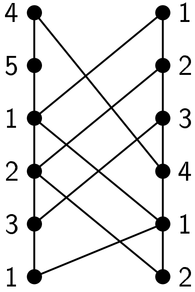

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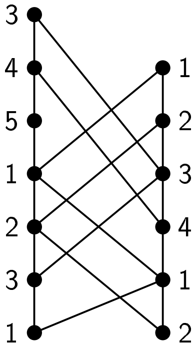

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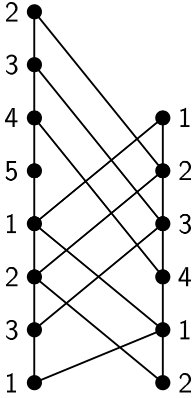

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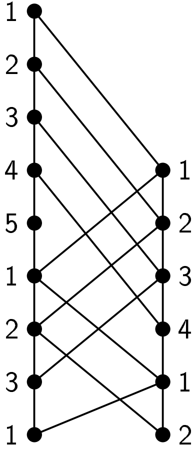

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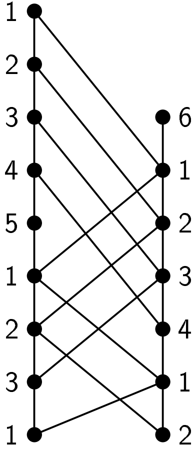

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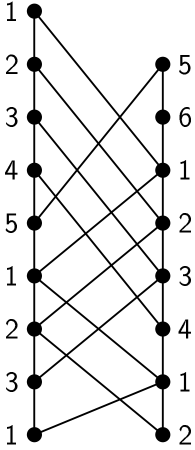

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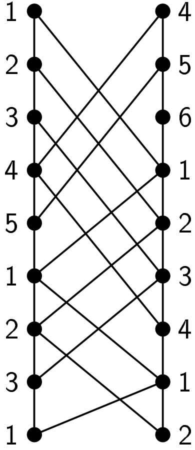

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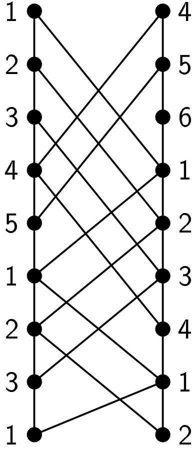

Remarks

- *First-Fit works well for some classes of posets (e.g. interval posets)*
- *is used as a subroutine in on-line algorithms that give subexponential bounds on* ocp(w).

first-fit

$f_Q(w)$ – the maximum number of chains used by first-fit on $Q-$free posets of width w

on-line chain partitioning

Theorem (Bosek, Krawczyk 2009)

$$\mathrm{ocp}(w) \leqslant w^{13 \log w}$$

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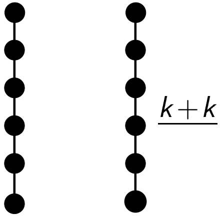
Remarks

- *First-Fit on $\underline{2w+2w}-$free posets is used as a subroutine*
- *First-Fit works efficiently on $\underline{k+k}-$free posets!*



First-Fit for $\underline{k+k}$-free posets

Upper bounds on $f_{\underline{k+k}}(w)$:

- $3kw^2$, Bosek, Krawczyk, Szczypka 2008
- $8k^2w$, Joret, Milans 2010
- $16kw$, Dujmović, Joret, Wood 2011 (G. Joret talk).

Lower bounds on $f_{\underline{k+k}}(w)$:

- $(k-1)(w-1)$, Dujmović, Joret, Wood 2011.

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Theorem (2011)

$$\operatorname{ocp}(w) \leqslant w^{3+6.5\log w}$$

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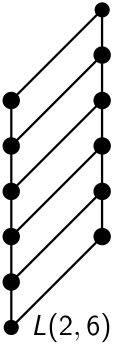
First-Fit on $L(2, 2w^2)-$*free posets is used as a subroutine*



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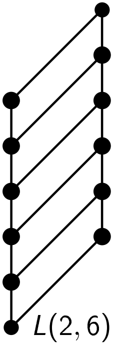
Proof:

- $\mathrm{ocp}(w) \leqslant w \cdot f_{L(2,2w^2)}(w)$,
  Bosek, Krawczyk 2009
- $f_{L(2,m)}(w) \leqslant w^{2.5\log w + 2\log m}$,
  Kierstead, M.Smith 2011 (M. Smith talk)
- subexponential bound on $\mathrm{ocp}(w)$ is best possible in this approach, Bosek, Matecki 2012



$L(2, 6)$

question

Question (Joret, Milans 2010)

For which posets Q there is a function f_Q such that First-Fit partitions $Q-$free posets of width w into at most $f_Q(w)$ chains.

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Remarks

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  - $L(2,2)$: $f_{L(2,2)}(w) = w^2$ *(tight result: Matecki 2011; Kierstead, M.Smith 2011)*

main result

Theorem (Bosek, Matecki, Krawczyk 2010)

Let Q be a poset. There exists a function f_Q such that First-Fit partitions Q-free posets of width w into at most $f_Q(w)$ chains iff Q is a poset of width 2.

main result

Theorem (Bosek, Matecki, Krawczyk 2010)

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Remark

If First-Fit uses unlimited number of chains on a (countable) poset P of width w then P contains all width 2 posets.

idea of the proof - ladders (1)

Ladders:

- universal posets of width 2, parameterized by two variables

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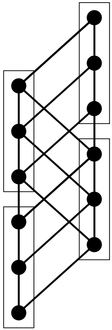
Lemma

For any poset Q of width 2 there exist sufficiently large s, t such that Q is contained in $L(s, t)$.



$L(4, 3)$

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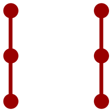
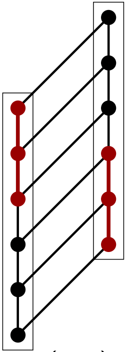


$3 + 3$



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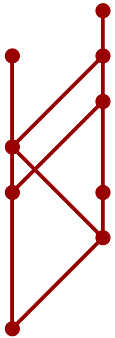
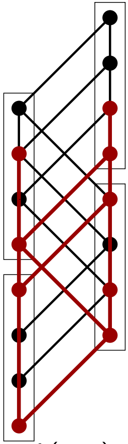


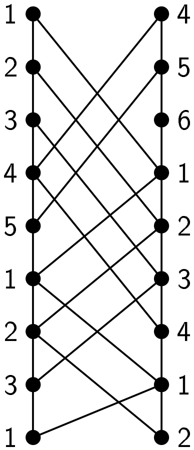


$L(4, 4)$

the Kierstead's example

Remark

The poset from the Kierstead's example must contain all posets of width 2.



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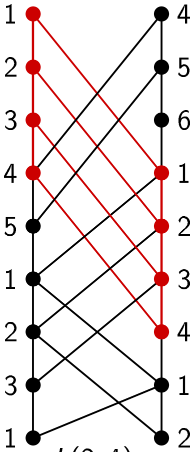

$L(2,4)$

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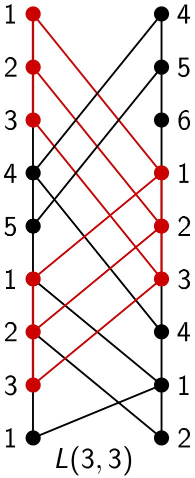
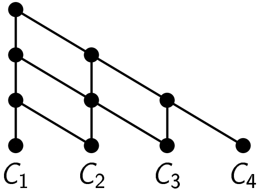


idea of the proof - wall

Definition

*Let $(P, \leqslant)$ be a poset. An ordered chain partition $C_1, C_2, \ldots, C_k$ of $(P, \leqslant)$ is a **wall** of size k if for any $x \in C_i$ and for any chain C_j, $j < i$, there is a point in C_j that is incomparable to x.*



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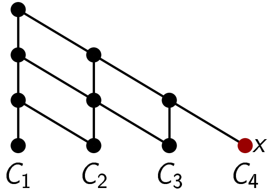

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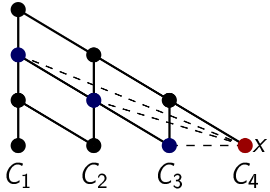

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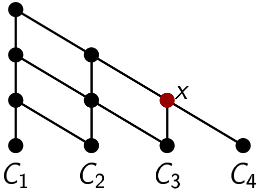

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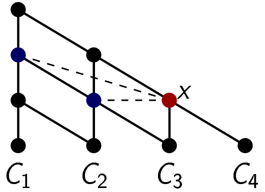

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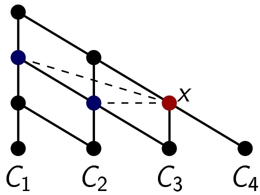


Remark

The maximum number of colors used by First-Fit on $(P, \leqslant)$ is equal to the maximum size of a wall in $(P, \leqslant)$.

- *First-Fit coloring of $(P, \leqslant)$ in the order $C_1 < C_2 < \ldots < C_k$ uses k colors,*
- *a partition $C_1, \ldots, C_k$ produced by first-fit is a wall in $(P, \leqslant)$.*

idea of the proof

We show that for every $s, t > 1$ the following holds:

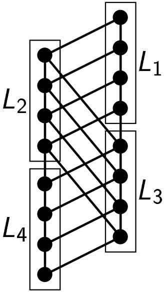
- any wall of width $w \geq 2$ and 'sufficiently large' size (i.e. first-fit uses a lot of colors) contains an $(s, t)-$ladder.

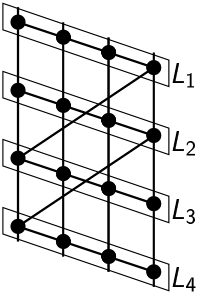
idea of the proof

We show that for every $s, t > 1$ the following holds:

- any wall of width $w \geq 2$ and 'sufficiently large' size (i.e. first-fit uses a lot of colors) contains an $(s, t)-$ladder.

- we 'are looking' for the 'wall-style' ladders





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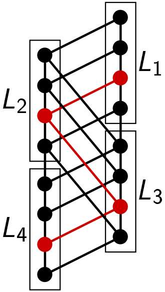

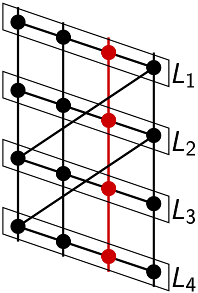

idea of the proof

First part:

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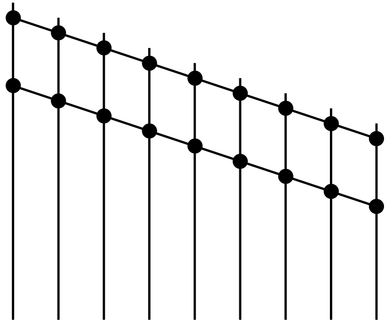
- every 'sufficiently large' wall of width $w \geq 2$ contains $L(2, t)$ ladder (M. Smith talk)



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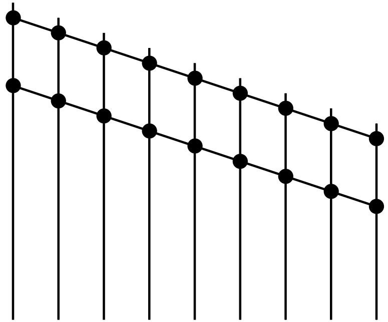

Second part:

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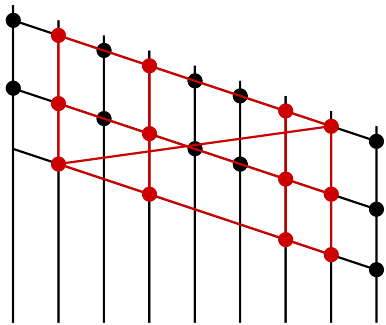
- every 'sufficiently large' wall of width $w \geq 2$ contains $L(2,t)$ ladder (M. Smith talk)

Second part:

- for every s, if $L(2,t)$ is large enough, then we may attach another level to $L(2,t)$ to get $L(3,s)$



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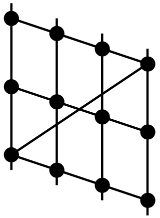
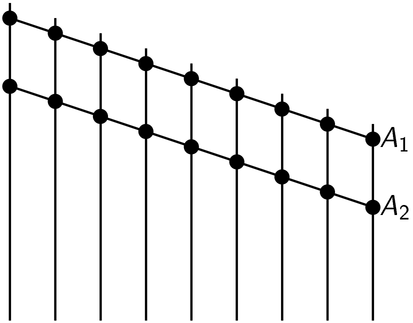


We focus on $L(3, 4)$.

idea of the proof



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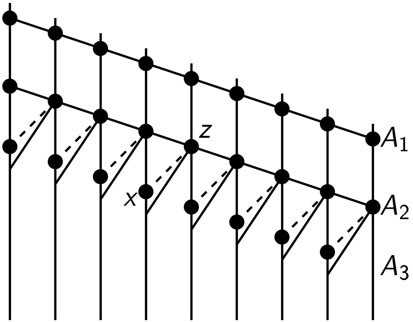
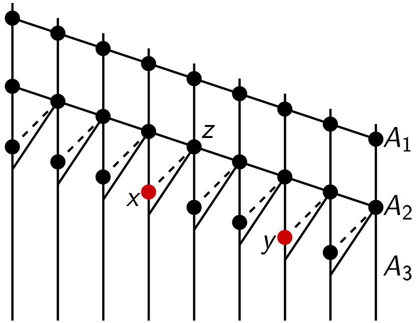


idea of the proof

- x, y - two points from A_3, x is to the left of y:

$$x || y \text{ or } x \geq y$$



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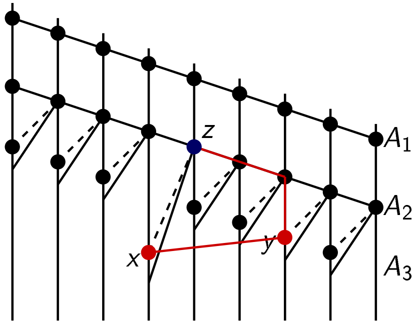

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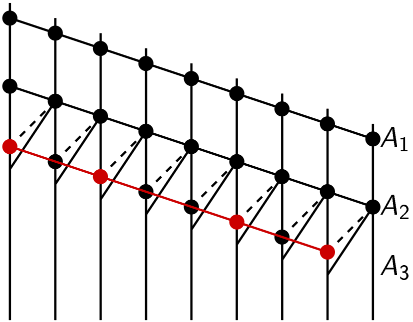

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- x, y - two points from A_3,
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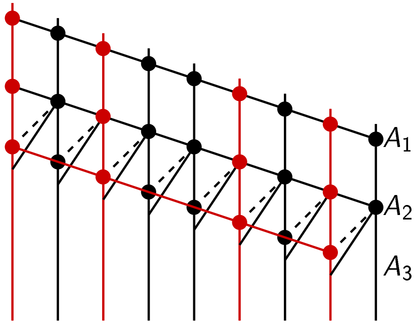

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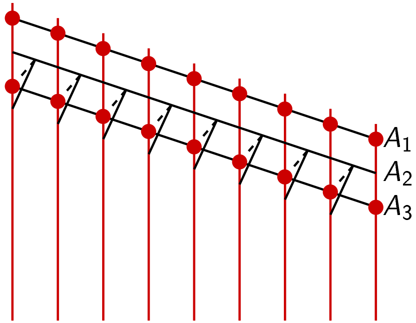
- C can be arbitrarily large!



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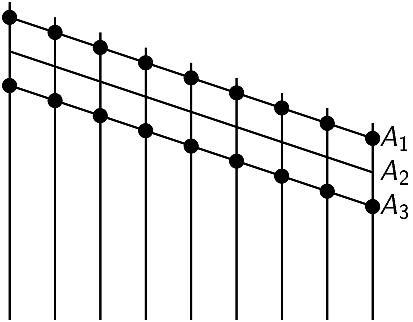

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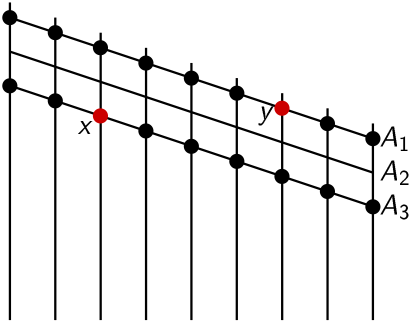

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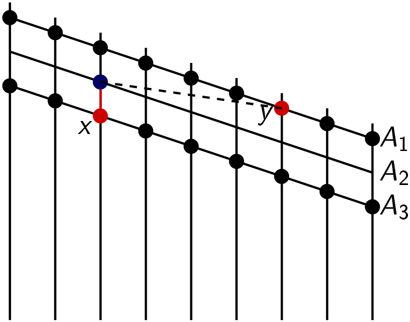

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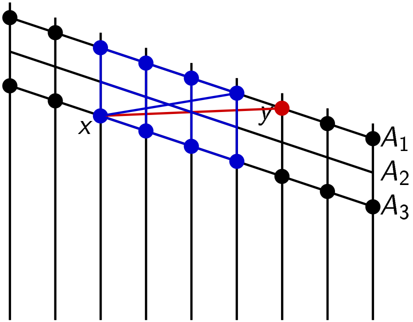

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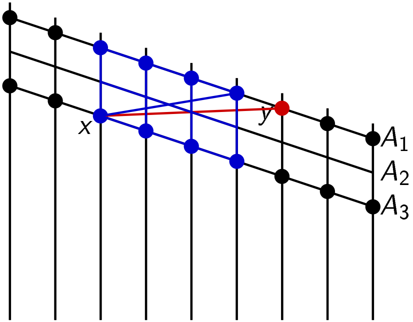

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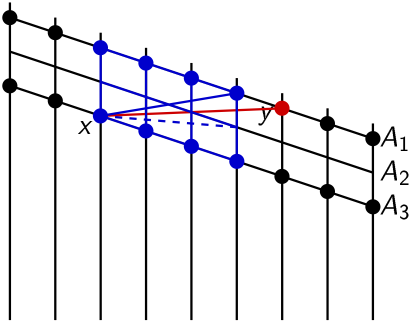

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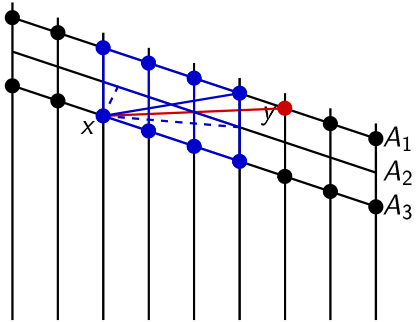

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- $x \in A_3$, $y \in A_1$, x is 'far' to the left of y:

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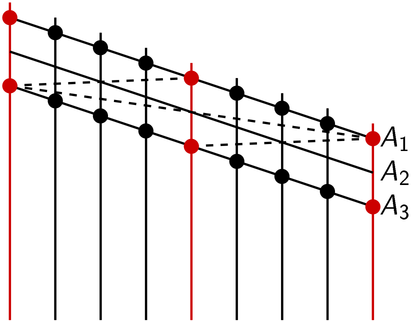
- choose every 4−th chain of the wall



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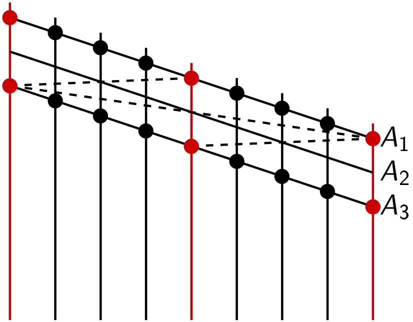
- chosen points from A_1 and A_3 form $L(2, *)$



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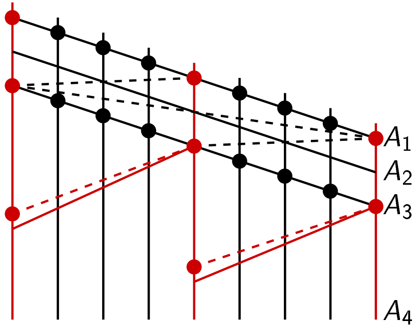

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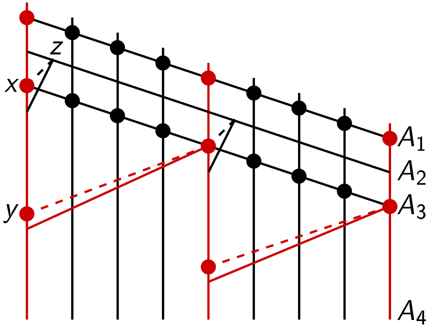

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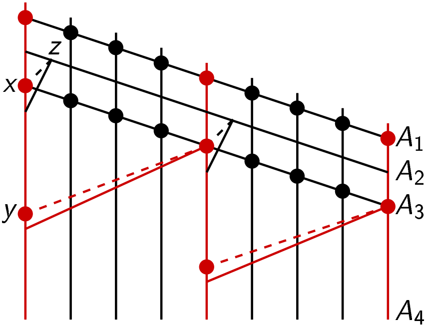
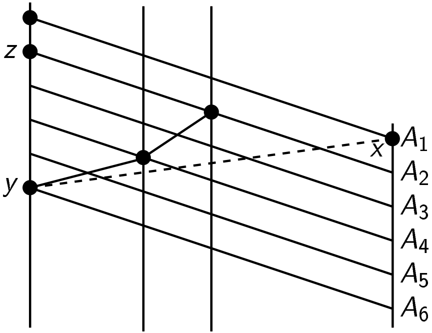
- each element in A_4 is below some element in A_2
- we construct the sets A_3, A_4, A_5, A_6, e.t.c.



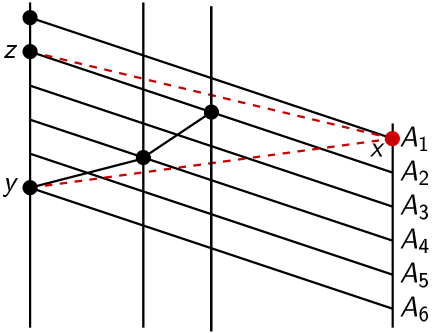
idea of the proof

- consider $L(2,*)$ formed by A_1 and A_6



idea of the proof

- consider $L(2, *)$ formed by A_1 and A_6
- x is incomparable to z



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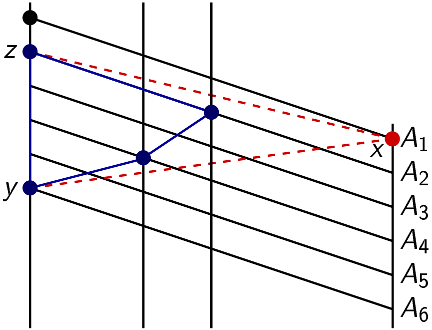

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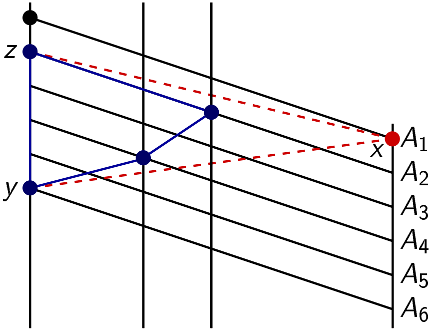

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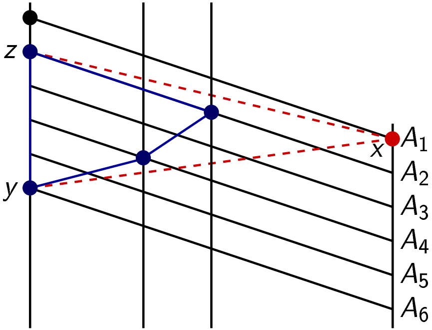
- consider $L(2,*)$ formed by A_1 and A_6
- x is incomparable to z
- x is incomparable to $y\Uparrow \cap z\Downarrow$
- $y\Uparrow \cap z\Downarrow$ induces a subwall of width $w-1$
- by induction, if the wall $y\Uparrow \cap z\Downarrow$ is large enough then it contains $L(3,4)$ ladder



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Open problems:

- find posets Q for which $f_Q(w) \leqslant poly(w)$

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Open problems:

- find posets Q for which $f_Q(w) \leqslant poly(w)$
- prove a subexponential bound on $f_{L(3,m)}$ and then for $f_{L(s,m)}$ (and hence for every Q of width 2)

Thank You!