1. \[ \begin{array}{c}
  \begin{array}{c}
  g \rightarrow h \\
  c \rightarrow d \rightarrow e \\
  b \rightarrow 3
  \end{array}
  \end{array} \]

4 a. This poset is not an interval order. Find four points which induce a copy of \(2 + 2\).
\[ c, e, d, b \]

4 b. The width of this poset is 4. Find a maximum antichain.
\[ \{ c, f, a, d \} \]

4 c. By inspection, find a partition into 4 chains. Give your answer by labeling the points on the diagram with the integers from \(\{1, 2, 3, 4\}\) so that all points with the same label form a chain.

2. \[ \begin{array}{c}
  \begin{array}{c}
  f \rightarrow \text{foo} \\
  b \rightarrow \text{bar} \\
  g \rightarrow \text{baz} \\
  e \rightarrow \text{qux} \\
  d \rightarrow \text{quux}
  \end{array}
  \end{array} \]

5 a. This poset is an interval order. It has 5 distinct down sets. Find them.
\[ \begin{array}{c}
  D_1^{D_2} \leq \{ c, d \} \leq \{ g, c, d \} \leq \{ e, g, c, d \} \leq \{ D_5 \}
  \end{array} \]

5 b. This poset also has 5 distinct up sets. Find them.
\[ \begin{array}{c}
  \{ a, g, f, b \} \geq \{ g, f, b \} \geq \{ f, b \} \geq \{ f \} \geq \{ \}
  \end{array} \]

5 c. Find the unique interval representation for this poset where every element is assigned an interval with integer endpoints from \(\{1, 2, 3, 4, 5\}\).
\[ \begin{array}{c}
  \Gamma(a) = [2, 5] \\
  \Gamma(b) = [4, 5] \\
  \Gamma(c) = [1, 2] \\
  \Gamma(d) = [1, 5] \\
  \Gamma(e) = [1, 4] \\
  \Gamma(f) = [5, 5] \\
  \Gamma(g) = [3, 3]
  \end{array} \]
3. Define an interval order $P$ with point set $X = \{a, b, c, d, e, f, g, h, j, k\}$ by the following interval representation.

![Diagram of interval order]

4. a. Use the First Fit algorithm to a partition of this poset into a minimum number of chains. Provide your answer by labeling the intervals in the diagram with positive integers so that all elements assigned the same integer form a chain.

4. b. Find a maximum antichain $A$ in this poset to provide a proof that your partition in part a is indeed minimum.

\{d, i, b, j, g, h\}

4. 10. a. Use the Greedy Algorithm described in class to find an euler circuit in this graph. Your answer should be given as a sequence of partial circuits starting with the trivial circuit (a). Proceed by taking the first vertex adjacent to a remaining edge and always taking the first available edge—using the alphabetic order.

\[(a) \rightarrow (a, b, d, e, a) \rightarrow (a, b, e, i, b, d, c, a) \rightarrow (b, e, i, b) \rightarrow (d, g, j, f, h, j, d, k, l, d) \rightarrow (a, b, e, i, b, d, g, j, f, h, j, d, k, l, e, d, c, a)\]

4. b. Show that the chromatic number of this graph is 3 by labeling each vertex in the diagram with an integer from \{1, 2, 3\} so that all vertices with the same label form an independent set.

4. c. Find a maximum clique in this graph

*Any triangle will do. For example, \{d, k, l\}*

4. d. Find a maximum cycle in this graph

\{a, b, d, c\}
In the space below, list in order the edges which make up a minimum weight spanning tree using, respectively Kruskal’s Algorithm (avoid cycles) and Prim’s Algorithm (build tree). For Prim, use vertex a as the root.

Kruskal’s Algorithm

\[
\begin{align*}
\text{c} \rightarrow \text{g} & : 23 \\
\text{d} \rightarrow \text{h} & : 29 \\
\text{f} \rightarrow \text{c} & : 31 \\
\text{c} \rightarrow \text{e} & : 32 \\
\text{c} \rightarrow \text{d} & : 37 \\
\text{b} \rightarrow \text{h} & : 38 \\
\text{a} \rightarrow \text{e} & : 47
\end{align*}
\]

Prim’s Algorithm

\[
\begin{align*}
\text{a} \rightarrow \text{e} & : 47 \\
\text{e} \rightarrow \text{c} & : 32 \\
\text{c} \rightarrow \text{g} & : 23 \\
\text{f} \rightarrow \text{c} & : 31 \\
\text{c} \rightarrow \text{d} & : 37 \\
\text{d} \rightarrow \text{h} & : 29 \\
\text{h} \rightarrow \text{b} & : 38
\end{align*}
\]

6.

\[a. \text{ Show that this graph is hamiltonian by listing the vertices in an order which forms a cycle of size 9.} \]

\[(a, c, e, h, i, d, g, b, f)\]

\[b. \text{ Explain why this graph has neither an euler circuit nor an euler path.} \]

\[\text{It has more than two vertices of odd degree.}\]
A data file digraph.data.txt has been read for a digraph whose vertex set is \([7]\). The weights on the directed edges are shown in the matrix below. Apply Dijkstra’s algorithm to find the distance from vertex 1 to all other vertices in the graph. Also, for each \(x\), find a shortest path from 1 to \(x\).