1. Consider the 26-element set consisting of the capital letters of the English alphabet: \{A, B, C, \ldots, Z\}.
   a. How many strings of length 12 can be formed if repetition of symbols is permitted?

   b. How many strings of length 12 can be formed if repetition of symbols is not permitted?

   c. How many strings of length 12 can be formed using exactly four X’s, three Y’s and five Z’s?

   d. How many strings of length 12 can be formed using exactly four X’s, three Y’s and five Z’s if the three Y’s are required to occur consecutively in the string?

2. How many lattice paths from (0, 0) to (24, 31) do not pass through (15, 19)?

3. How many integer valued solutions to the following equations and inequalities:
   a. \(x_1 + x_2 + x_3 = 42\), \(x_1, x_2, x_3 > 0\).

   b. \(x_1 + x_2 + x_3 = 42\), \(x_1, x_2, x_3 \geq 0\).

   c. \(x_1 + x_2 + x_3 < 42\), \(x_1, x_2, x_3 > 0\).

   d. \(x_1 + x_2 + x_3 \leq 42\), \(x_1, x_2, x_3 \geq 0\).

   e. \(x_1 + x_2 + x_3 = 42\), \(x_1, x_3 > 0, x_2 \geq 7\).

   f. \(x_1 + x_2 + x_3 = 42\), \(x_1, x_3 > 0, 0 < x_2 \leq 6\).
4. Use the Euclidean algorithm to find \( d = \gcd(420, 245) \).

5. Use your work in the preceding problem to find integers \( a \) and \( b \) so that \( d = 420a + 245b \).

6. For a positive integer \( n \), let \( s_n \) count the number of ternary strings of length \( n \) that do not contain 102 as a substring. Note that \( s_1 = 3, \ s_2 = 9 \) and \( s_3 = 26 \). Develop a recurrence relation for \( s_n \) and use it to compute \( s_4, s_5 \) and \( s_6 \).
7. Let $t_n$ denote the number of ways to tile a $2 \times n$ checkerboard using tiles of the two shapes shown on the white board. One of the shapes is an “L” with a total area of 3. As illustrated, this shape can be used “forwards” and “backwards” but not upside down. The other shape is a $2 \times 1$ strip and it can be used vertically or horizontally. Note that $t_1 = 1$, $t_2 = 2$ and $t_3 = 3$. Develop a recurrence for $t_n$ and use it to compute $t_4$, $t_5$ and $t_6$.

8. Find the coefficient of $x^4y^7z^{24}$ in $(6x - 5y + 8z^2)^{23}$

1. $P(7, 3) = 1024$.
2. $C(7, 3) = 35$.
3. If 57 pigeons are placed in 7 holes, then there is some hole with at least 9 pigeons.
4. If $f(n) = 865n + 90 \log n$, and $g(n) = 3n + 7$, then $f(n) = O(g(n))$.
5. If $f(n) = 865n + 90 \log n$, and $g(n) = 3n^2 + 7$, then $f(n) = o(g(n))$.
6. $\log n = o(\sqrt{n})$, $\sqrt{n} = o(n)$, $n = o(n^3)$, $n^3 = o(2^n)$, $2^n = o(2^{n^2})$ and $2^{n^2} = o(2^{2^n})$.
7. A recursive permutation tiles non-distinct pigeons with a certificate that can be enumerated but not verified.