1. In the space to the right, use the algorithm developed in class (always proceed to the lowest legal vertex) to find an Euler circuit in the graph $G$ shown below (use node 1 as root):

![Graph G](image)

2. For the graph below,

![Graph](image)

(a) Find a maximal clique of size 2.

(b) Find a maximal clique of size 3.

(c) Find a maximal clique of size 4.

(d) Find an induced cycle of size 5.

(e) Show that $\chi(G) \leq 4$ by producing a proper coloring using the elements of $\{1, 2, 3, 4\}$ as colors. You may write directly on the figure.
3. In the space to the right, verify Euler’s formula for the following graph.

![Graph](image)

4. In the space to the right, explain why the following graph does not have an Euler circuit—
   *without* carrying out the algorithm from Problem 1. Then show that it has a hamiltonian cycle
   starting (1, 6, 8, 3). You may either complete the sequence or darken the appropriate edges on the
   figure.

![Graph](image)

5. Consider the poset shown below (two copies are shown).

![Poset](image)

(a) Find the set of maximal elements.

(b) Find the set of minimal elements.

(c) Find all points comparable with $h$.

(d) Find all points incomparable with $e$.

(d) Find all points covered by $h$.

(e) Find all points which cover $h$. 

______________

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(f) Find a maximal chain of size 3.

(g) Find a maximal antichain of size 2.

(h) This poset is not an interval order. Find, by inspection, four points which form a copy of $2 + 2$.

(i) Recursively strip off the minimal elements and find the height $h$ of the poset. Also find a partition of the poset into $h$ antichains. You may provide your answer by labelling the points in the figure on the left with integers from $\{1, 2, \ldots, h\}$ so that all points labelled with the same integer form an antichain.

The height $h$ is ______ and __________________ is a maximum chain.

(j) Find, by inspection, the width $w$ of the poset. Also, find a partition of the poset into $w$ chains. You may provide your answer by labelling the points in the figure on the right with integers from $\{1, 2, \ldots, w\}$ so that points labelled with the same integer form a chain.

The width $w$ is ______ and __________________ is a maximum antichain.

6. Shown below is the diagram of an interval order. Use the algorithm taught in class to find an interval representation by computing the down-sets and up-sets in the space provided. Then use the First Fit coloring algorithm to find the width $w$ and a partition of the poset into $w$ chains. Also, find a maximum antichain.

\[
\begin{align*}
D(a) &= \\
D(b) &= \\
D(c) &= \\
D(d) &= \\
D(e) &= \\
D(f) &= \\
D(g) &= \\
D(h) &= \\
\end{align*}
\]

\[
\begin{align*}
U(a) &= \\
U(b) &= \\
U(c) &= \\
U(d) &= \\
U(e) &= \\
U(f) &= \\
U(g) &= \\
U(h) &= \\
\end{align*}
\]

The width $w$ is ______ and __________________ is a maximum antichain.
7. Let $2^{13}$ be the poset consisting of all subsets of $\{1, 2, 3, \ldots, 13\}$, ordered by inclusion.

a. What is the height of this poset? ____________

b. What is the width of this poset? ____________

c. How many maximal chains does the poset have? ____________

d. How many maximal chains in this poset pass through the set $\{2, 4, 9, 11\}$? ____________

8. Write the inclusion-exclusion formula for the number $d_n$ of derangements of $\{1, 2, \ldots, n\}$. Then use this formula to calculate $d_6$.

9. Write the inclusion-exclusion formula for the number $S(n, m)$ of surjections from $\{1, 2, \ldots, n\}$ to $\{1, 2, \ldots, m\}$. Then use this formula to calculate $S(5, 3)$.

10. Note that $252 = 2^2 \cdot 7 \cdot 9$. Use this information and the inclusion-exclusion formula to determine $\phi(252)$, where $\phi$ is the Euler $\phi$-function studied in class.

1. The number of labelled trees with vertex set \{1, 2, \ldots, n\} is \(\binom{n}{[n/2]}\).
2. There is a graph with 21 vertices in which no two vertices have the same degree.
3. There is a connected graph with 21 vertices and 25 edges which has an euler circuit.
4. There is a connected graph with 21 vertices and 25 edges which does not have an euler circuit.
5. There is a connected graph with 21 vertices and 25 edges which has a hamiltonian cycle.
6. There is a connected graph with 21 vertices and 25 edges which does not have a hamiltonian cycle.
7. There is a connected graph with 21 vertices and 110 edges which is not hamiltonian.
8. There is a connected planar graph with 21 vertices and 110 edges.
9. There is a poset on 21 points having width 4 and height 3.
10. There is a poset on 21 points having width 10 and height 8.
11. The number of deranged surjections of a transitive hamiltonian trail is a planar function and can be effectively computed using prime factors of even degree and odd behavior.