MATH 3012 Quiz 2, October 19, 2017 WTT

1. Two copies of the same planar graph $G$ are shown below.

![Graph Image]

a. Verify Euler’s formula for the graph $G$. You may mark on the left copy of $G$ if you find it convenient to do so.

b. Find three vertices of $G$ which form a clique of size 3.

c. Show that $\chi(G) = \omega(G) = 3$ by indicating a 3-coloring of $G$ on the right copy.

d. Explain why $G$ is not perfect by listing a sequence of vertices showing that $G$ contains an induced cycle of size 9.

2. Consider the following poset.

![Poset Image]

a. Find all points comparable to $g$.

b. Find all points which cover $g$.

c. Find all points which are covered by $g$.

d. Find a maximal chain of size 2.

e. Find a maximal chain of size 3.

f. Find the set of all maximal elements.

g. Find the set of all minimal elements.

h. Using the algorithm taught in class (recursively removing the set of minimal elements), find the height $h$ of the poset and a partition of $P$ into $h$ antichains. Also find a maximum chain. You may indicate the partition by writing directly on the diagam. The height $h$ is _____ and ______________________ is a maximum chain.
3. Find by inspection the width \( w \) of the following poset and find a partition of the poset into \( w \) chains. Also find a maximum antichain. You may indicate the partition by writing directly on the diagram.

![Diagram of a poset]

a. The width \( w \) is _____ and ___________ is a maximum antichain.

b. This poset is not an interval order. Find by inspection four points which form a copy of \( 2 + 2 \). _______________.

4. Shown below is the diagram of an interval order. Use the algorithm taught in class to find an interval representation by computing the down-sets and up-sets in the space provided. Then use the First Fit coloring algorithm to find the width \( w \) and a partition of the poset into \( w \) chains. Also, find a maximum antichain.

![Diagram of an interval order]

\[
D(a) = \quad U(a) = \\
D(b) = \quad U(b) = \\
D(c) = \quad U(c) = \\
D(d) = \quad U(d) = \\
D(e) = \quad U(e) = \\
D(f) = \quad U(f) = \\
D(g) = \quad U(g) = \\
\]

The width \( w \) is ________ and ______________ is a maximum antichain.
5. True–False. Mark in the left margin.

1. There is a graph $G$ with $\omega(G) = 2$ and $\chi(G) = 100$.
2. There is a graph $G$ with $\omega(G) = 3$ and $\chi(G) = 100$.
3. There is a planar graph $G$ with $\omega(G) = 2$ and $\chi(G) = 100$.
4. If $\chi(G) = 2$, then $G$ is perfect.
5. If $\chi(G) = 3$, then $G$ is perfect.
6. There is a graph $G$ with 24 vertices and 100 edges such that $\chi(G) = \omega(G) = 2$.
7. There is a planar graph with 24 vertices and 100 edges.
8. There is a poset with 3209 points having width 79 and height 39.
9. There is a poset with 3209 points having width 97 and height 93.
10. When $n \geq 3$, the shift graph $S_n$ contains a triangle.