Quiz 2, Math 2552

PLEASE PRINT YOUR NAME CLEARLY IN ALL CAPITAL LETTERS

First Name ___________________  Last Name ___________________

GTID Number: ___________________

Student GT Email Address: ___________________@gatech.edu

Section Number (D1, D2 or D3) _______  TA Name ___________________

Student Instructions

• Show your work and justify your answers for all questions unless stated otherwise.
• Organize your work in a reasonably neat and coherent way.
• Calculators, notes, cell phones, books are not allowed.
• Use dark and clear writing: your exam will be scanned into a digital system.
• Exam pages are double sided. Be sure to complete both sides.
• Leave a 1 inch border around the edges of exams.
• Any work done on scratch paper will not be collected and will not be graded.
1. (10 points) Consider the differential equation \( \frac{dy}{dt} = (y - 4)^2(y^2 - 1) \).

   (a) (2 points) Find all equilibria.

   (b) (4 points) Sketch the phase line.

   (c) (4 points) Determine whether each equilibrium is stable, semi-stable or unstable.
2. (10 points) Consider the 2nd order equation with initial value:
\[
\begin{align*}
(t + 1)y''(t) + e^t y'(t) + 3y(t) &= t \sin(t), \\
y(0) &= -2, \quad y'(0) = -2.
\end{align*}
\]

(a) (6 points) Transform this equation into a system of first order equations, and express it into matrix form.

(b) (2 points) Is the first order system you obtained homogeneous? Is it autonomous?

(c) (2 points) Write down the largest interval in which a unique solution of the IVP is guaranteed to exist.
3. (10 points) Solve the initial value problem \( \frac{d\vec{x}}{dt} = A\vec{x}, \vec{x}(0) = \begin{pmatrix} 3 \\ -3 \end{pmatrix} \), where \( A \) is a 2\( \times \)2 constant matrix having:

- an eigenvalue \( \lambda_1 = 4 \) with eigenvector \( \vec{u}_1 = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \),
- an eigenvalue \( \lambda_2 = -1 \) with eigenvector \( \vec{u}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \).