Section 1.1 : Mathematical Models and Solutions

Chapter 1 : Introduction

Math 2552 Differential Equations

Section 1.1

Topics
We will cover these topics in this section.

1. Mathematical Models and Direction Fields
2. Newton's Law of Cooling

Objectives
For the topics covered in this section, students are expected to be able to do the following.

1. Apply an exponential growth/decay model to solve and analyze first order DEs

Example: Newton's Law of Cooling

Suppose a system under observation is an object at temperature $u(t)$, at time t, and is located in an environment with constant ambient temperature T.

Newton's Law of Cooling: the rate of change of the temperature of an object is negatively proportional to the difference between $u(t)$ and T.

$$\frac{d}{dt}u = -k(u - T) \tag{1}$$

Here, u is an **unknown**, and k and T are **parameters** of the system.

Equation (1) is an example of a **differential equation**.

Definition: Differential Equation

A **differential equation** is an equation involving a function and its derivatives.

Solution to a DE

- A **solution** of a DE is a differentiable function that satisfies that DE on some interval.
- To determine whether a function is a solution to a given DE, what can we do?

Example: Verify that $Ce^{-kx} + T$, $C \in \mathbb{R}$, is a solution to the DE

$$\frac{d}{dt}u = -k(u - T)$$

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Dynamical Systems

Newton's Law is one example of an equation that describes a **dynamical system**.

A dynamical system is composed of:

- A **system**: Which means that we are observing a phenomenon which behaves according to a set of laws.
- The phenomenon may be mechanical, biological, social, etc.
- **Dynamics**: the system evolves in time.

It is our task to **predict** and **characterize** (as much as possible) the long-term behavior of the dynamical system and how it changes. This leads us to the use of derivatives and the methods we explore for the rest of the course.