Section 1.1 : Mathematical Models and Solutions

Chapter 1 : Introduction

Math 2552 Differential Equations
Section 1.1

**Topics**
We will cover these topics in this section.

1. Mathematical Models and Direction Fields
2. Newton’s Law of Cooling

**Objectives**
For the topics covered in this section, students are expected to be able to do the following.

1. Apply an exponential growth/decay model to solve and analyze first order DEs
Example: Newton’s Law of Cooling

Suppose a system under observation is an object at temperature \( u(t) \), at time \( t \), and is located in an environment with constant ambient temperature \( T \).

Newton’s Law of Cooling: the rate of change of the temperature of an object is negatively proportional to the difference between \( u(t) \) and \( T \).

\[
\frac{d}{dt} u = -k(u - T) \tag{1}
\]

Here, \( u \) is an unknown, and \( k \) and \( T \) are parameters of the system.

Equation (1) is an example of a differential equation.

Definition: Differential Equation

A differential equation is an equation involving a function and its derivatives.
Solution to a DE

- A **solution** of a DE is a differentiable function that satisfies that DE on some interval.
- To determine whether a function is a solution to a given DE, what can we do?

**Example:** Verify that \( Ce^{-kx} + T, C \in \mathbb{R} \), is a solution to the DE

\[
\frac{du}{dt} = -k(u - T)
\]
Newton’s Law is one example of an equation that describes a dynamical system.

A dynamical system is composed of:

- **A system**: Which means that we are observing a phenomenon which behaves according to a set of laws.
- The phenomenon may be mechanical, biological, social, etc.
- **Dynamics**: the system evolves in time.

It is our task to **predict** and **characterize** (as much as possible) the long-term behavior of the dynamical system and how it changes. This leads us to the use of derivatives and the methods we explore for the rest of the course.