Section 1.3 : Definitions, Classifications, Terminology

Chapter 1 : Introduction

Math 2552 Differential Equations
Topics
We will cover these topics in this section.

1. Classification of ODEs
2. Standard Form

Objectives
For the topics covered in this section, students are expected to be able to do the following.

1. Classify differential equations
2. Convert a first order linear ODE into standard form
Section 1.3 introduces some of the ways we can classify differential equations.

Classification allows us to determine which methods we can use to solve a DE.
Ordinary and Partial Differential Equations

- **Ordinary Differential Equation (ODE)**: the functions in the DE depend on only one a single independent variable.

- **Partial Differential Equation (PDE)**: the functions in the DE depend on more than one independent variable.

This course focuses on ODEs. PDEs are the focus of more advanced courses that are offered by the School of Math.

**Examples**

The heat equation, \( \frac{\partial}{\partial t} u(x, t) = D \frac{\partial^2 u}{\partial x^2} \) is an example of a PDE.

Newton’s Law of Cooling, \( \frac{d}{dt} u(t) = -k(u - T) \), is an example of an ODE.
The Order of an ODE

- The **order** of an ODE is the highest degree derivative which appears in the equation.
- This course mostly focuses on first and second order ODEs.

**Example:** What is the order of the ODE $u^{(3)} + 2e^tu'' + uu' = t^4$?
Linear ODEs

An $n^{th}$ order linear ODE takes the form

$$\sum_{k=0}^{n} a_k u^{(n-k)}(t) = a_0(t)u^{(n)}(t) + a_1(t)u^{(n-1)}(t) + \ldots + a_n(t)u(t) = g(t)$$

The coefficients $a_0(t)$, $a_1(t)$, $\ldots$, $a_n(t)$ and $g(t)$ are given, $u(t)$ is unknown.

If $g(t) = 0$, we say that the linear ODE is homogeneous.

A DE that is not of this form is non-linear. The coefficients may be non-linear with respect to $t$. 
Standard Form of a First Order DE

The general first order linear equation is of the form

\[ a_0(t) \frac{dy}{dt} + a_1(t)y = h(t) \quad [*] \]

Again, \( a_0, a_1 \) and \( h \) are given, and \( y \) is the unknown.

**Definition: Standard Form, 1st Order Linear ODE**

If \( a_0(t) \neq 0 \), we can divide by \( a_0(t) \) and put \([*]\) in the form

\[ \frac{dy}{dt} + p(t)y = g(t) \]

where \( p(t) := \frac{a_1(t)}{a_0(t)} \) and \( g(t) := \frac{h(t)}{a_0(t)} \). This the **Standard Form**
Examples

Which of the following DEs are linear? What is the order of the DEs?

1. \( t^3 \frac{d^2 y}{dt^2} + \sin(t) \frac{dy}{dt} = y \)
2. \( y \frac{dy}{dt} = t \)
3. \( (1 + t) \frac{d^3 y}{dt^3} = \sin(t + y) \)
Classification allows us to determine which methods we can use to solve the DE.

We can classify an ODE based on
- the order of the equation
- linearity
- whether the DE is homogeneous

We will introduce other ways to classify DEs.