Section 2.2 : Linear Equations: Integrating Factors

Chapter 2 : First Order Differential Equations

Math 2552 Differential Equations
Section 2.2

**Topics**
We will cover these topics in this section.

1. Solving a first order linear ordinary differential equations using a procedure that uses an integrating factor

**Objectives**
For the topics covered in this section, students are expected to be able to do the following.

1. Convert differential equations into standard form
2. Classify differential equations as linear
3. Solve first order linear ODEs using an integrating factor
• We explore a method to solve a first order **linear** ODE in this section.

• Recall that an ODE is in **standard form** if it can be written as

\[ \frac{dy}{dt} + p(t)y(t) = y(t) \]

• We will solve an equation in standard form on the next slide, and then discuss the solution method more generally.
Example 1

Consider the differential equation $ty' + 2y = 4t$ for $t \geq 0$.

1. Is this differential equation separable?
2. Solve the differential equation.
The Integrating Factor

Given a linear first order ODE in standard form,

\[
\frac{dy}{dt} + p(t)y = g(t)
\]  

(1)

Multiply by the **integrating factor** \( \mu(t) = e^{\int p \, dt} \).

\[
\mu(t) \frac{dy}{dt} + p(t)\mu(t)y = \mu(t)g(t)
\]  

(2)

\( \mu \) is constructed so that the left-hand side is a derivative of a product.

\[
\frac{d}{dt} (\mu(t)y(t)) = \mu(t) \frac{dy}{dt} + p(t)\mu(t)y
\]  

(3)

We construct \( \mu \) so that the left-hand side of (2) is the derivative of \( \mu(t) y \).
This gives us a procedure for solving a first order linear DE.

\[
\frac{dy}{dt} + p(t)y(t) = g(t)
\]

1. Convert given DE to standard form (if necessary).
2. Calculate integrating factor \( \mu(t) = e^{\int p(t) \, dt} \)
3. Multiply DE by \( \mu \), express DE in the form \( [\mu(t)y]' = \mu g \).
4. Integrate
5. Solve for \( y \)

We will explore more examples of solving linear equations in the next section.