Section 2.3 : Modeling with First Order Equations

Chapter 2 : First Order Differential Equations

Math 2552 Differential Equations

Section 2.3

Topics

We will cover these topics in this section.

1. Solving a first order linear ordinary differential equations using a procedure that uses an integrating factor

Objectives

For the topics covered in this section, students are expected to be able to do the following.

1) Construct a differential equation to model a real-world situation.
2) Solve the differential equation so that we can interpret its solution to characterize a system.
3) Analyze mathematical statements and solutions of differential equations.

Example 1: Water Tank

A tank initially contains 40 pounds of salt dissolved in 600 gallons of water. Starting at time $t = 0$, water that contains $1/2$ pound of salt per gallon is poured into the tank at the rate of 4 gal/min and the mixture is drained from the tank at the same rate.

a) Construct a differential equation for $Q(t)$, which is the number of pounds of salt in the tank at time $t > 0$.

b) Solve the DE to determine an expression for $Q(t)$.

c) After a long period of time, what happens to the concentration of salt in the tank?

Mathematical Modeling

The process we used in the previous example roughly followed this process.

1) Construct a differential equation to model a real-world situation.
2) Solve the differential equation so that we can interpret its solution to characterize a system.
3) Analyze mathematical statements and solutions of differential equations.

The above process is used throughout this course, and so are captured in the course learning objectives that are stated in the syllabus.

Example 2: Population Model

The world population in 2018 is roughly 7.6 billion.

a) The world population is increasing at a rate of 1.2% per year. If the growth rate remains fixed at 1.2%, how long will it take for the population of the world to reach 20 billion people?

b) Assume the earth cannot support a population beyond 20 billion people. If the population growth rate is **also** proportional to the difference between how close the world population is to this limiting value, what is the expression that gives the world population as a function of time?