Section 3.5 : Repeated Eigenvalues

Chapter 3 : Systems of Two First Order Equations

Math 2552 Differential Equations
Topics
We will cover these topics in this section.

1. Systems of two first order linear DEs for the repeated eigenvalue case

Objectives
For the topics covered in this section, students are expected to be able to do the following.

1. Solve first order homogeneous linear systems that have repeated eigenvalues
2. Sketch component plots and phase portraits of linear systems of differential equations for repeated eigenvalues
Object Motion

The motion of an object moving in the $xy$-plane is given by $\mathbf{r}'(t)$, where

$$
\mathbf{r}'(t) = \begin{pmatrix}
x(t) \\
y(t)
\end{pmatrix}
$$

The velocity of the object is constrained by

$$
\frac{dx}{dt} = -x + ky 
$$

(1)

$$
\frac{dy}{dt} = -y
$$

(2)

Assume $k \in \mathbb{R}$. At time $t = 0$, our object is located at $\mathbf{r}(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

Take a few minutes on your own to solve this initial value problem. Create a rough sketch of the phase portrait. How your answer depend on the value of $k$?

Compare your answers with someone sitting nearby.
Example

Construct the general solution to the system.

$$\vec{x}' = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} \vec{x}$$