Section 4.6 : Forced Vibrations, Frequency Response, Resonance

Chapter 4 : Second Order Equations

Math 2552 Differential Equations
Topics
We will cover these topics in this section.

1. Forced spring-mass systems

Objectives
For the topics covered in this section, students are expected to be able to do the following.

1. Characterize a spring-mass system using the concepts of beats, resonance, transient solution, and steady-state solution

Students are not expected to solve questions regarding frequency response and gain.
Transient and Steady-State

From our undetermined coefficients lecture, we saw the damped, forced system

$$y'' + 3y' + 2y = \sin t$$

It has the solution

$$y = c_1 e^{-2t} + c_2 e^{-t} + 0.1 \sin t - 0.3 \cos t$$

The homogeneous solution corresponds to the **transient solution** and the particular solution corresponds to the **steady-state** solution.
Participation Activity: Worksheet

Depending on how much time we have, we will run examples in this lecture as a participation activity.

- Please work in groups of ________________
- Each group submits one sheet of paper
- Instructor has paper you can use
- Print full names and email addresses on cover
- Every student in a group gets the same grade
- Grading scheme per question:
  - 0 marks for no work or working alone
  - 1 mark for starting the problem or for a final answer with insufficient justification
  - 2 marks for a complete solution
Consider the spring-mass system

\[ y'' + \omega_0^2 y = F_0 \cos(\omega t), \quad y(0) = 0, \quad y'(0) = 0, \quad F_0 > 0 \]

Constants \( \omega_0 \) and \( \omega \) are positive. Solve the DE for the cases:

1. \( \omega \neq \omega_0 \)
2. \( \omega = \omega_0 \)