Section 6.3: Homogeneous Linear Systems with Constant Coefficients

Chapter 6: Systems of First Order Linear Equations

Math 2552 Differential Equations
Topics
We will cover these topics in this section.

1. Possible forms of the solution to a Homogeneous Linear Systems with Constant Coefficients

Objectives
For the topics covered in this section, students are expected to be able to do the following.

1. Express the fundamental set of solutions, and the general solution to $\vec{x}' = A\vec{x}$ for nondefective $A$. 
Motivation and Review

Key question in this section: what is the general form of the solution, and the fundamental set of solutions to a linear, first order, constant coefficient, system of DEs?

Our questions require the use of concepts from linear algebra.

a) What is the geometric multiplicity of an eigenvalue?

b) What is the algebraic multiplicity of an eigenvalue?

Example: $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$
Three Cases

Consider the linear, first order, constant coefficient system,

$$\vec{x}' = P\vec{x}, \quad \vec{x} = \vec{x}(t), \quad P \in \mathbb{R}^{n \times n}.$$ 

We assume solutions of the form

$$\vec{x}(t) = e^{\lambda t}\vec{v}.$$ 

Our eigenvalue problem has three cases:
**A Nondefective, Real Eigenvalues**

**Theorem**

If \((\lambda_1, \vec{v}_1), \ldots (\lambda_n, \vec{v}_n)\) are eigenpairs for \(n \times n\) matrix \(A\), and \(A\) has \(n\) linearly independent eigenvectors \(\vec{v}_1, \ldots, \vec{v}_n\), then \(\vec{x}' = A\vec{x}\) has:

- a general solution:

- a fundamental set of solutions:

Note: the eigenvalues need not be distinct.
Example

Determine the general solution to

\[ \vec{x}' = P \vec{x}, \quad P = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ 1 & -1 & 2 \end{pmatrix} \]

The eigenvalues of \( P \) are 0 and 1.