Section 6.4 : Nondefective Matrices with Complex Eigenvalues

Chapter 6 : Systems of First Order Linear Equations

Math 2552 Differential Equations

Section 6.4

Topics

We will cover these topics in this section.

1. First order linear homogeneous systems with complex eigenvalues.

Objectives

For the topics covered in this section, students are expected to be able to do the following.

1. Solve systems of constant coefficient, homogeneous, first order, linear differential equations with complex eigenvalues.

General Solution, Complex Eigenvalues

As we saw in a previous section, we have a general procedure for real-valued solution in the complex case.

1. Compute eigenvalues $\lambda = \alpha \pm i\beta$
2. Compute eigenvector, $\vec{v}$, for $\lambda = \alpha + i\beta$
3. Set $\vec{v} = \vec{a} + i\vec{b}$
4. General solution is $\vec{x}(t) = c_1\vec{u} + c_2\vec{w}$, where

$$\vec{u} = e^{\alpha t}(\vec{a}\cos\beta t - \vec{b}\sin\beta t)$$
$$\vec{w} = e^{\alpha t}(\vec{a}\sin\beta t + \vec{b}\cos\beta t)$$

You may want to memorize the above process and equations.

Particle Motion

The motion of a moving object is described by the system below.

$$\frac{dx}{dt} = -kx \tag{1}$$

$$\frac{dy}{dt} = -z \tag{2}$$

$$\frac{dz}{dt} = y \tag{3}$$

Assume $k > 0$. At time $t = 0$, our particle is located at the point $(x, y, z) = (1, 1, 0)$.

Take a few minutes on your own to solve this initial value problem. Express your solution in terms of real valued functions, and create a rough sketch of the particle motion.

Compare your answers with someone sitting nearby.

Expectations for Quizzes and Exams

- In the previous example, we sketched the solution curve passing through $(1, 1, 0)$ by hand. In general, we need to graph the curve using a computer. On quizzes/midterms, students are expected to sketch solution curves and phase portraits for two dimensional systems, but not for three.
- For three dimensional systems, students will be given the eigenvalues, or the coefficient matrix will have a lot of zeros so that the eigenvalues can be determined quickly.