Section 7.4 : Predator-Prey Equations

Chapter 7 : Systems of First Order Linear Equations

Math 2552 Differential Equations
Section 7.4

**Topics**
We will cover these topics in this section.

1. Predator-prey equations

**Objectives**
The objectives for this section are the same as those for 7.2. This section applies what we learned in 7.2 to specific cases.
Now suppose that we have two species that interact directly with other so that

- $x(t)$ and $y(t)$ are their populations
- $y$ preys on $x$
- in absence of prey, $y$ dies out, so $y' = -Cy$, $C > 0$
- in absence of predator, $x' = Ax$, $A > 0$
- the number of encounters is proportional to $xy$

These constraints give us

$$\frac{dx}{dt} = Ax - \alpha xy \quad (1)$$
$$\frac{dy}{dt} = -Cy + \gamma xy \quad (2)$$

These are the Lotka - Volterra equations. Students are not expected to memorize their general form.
Example

For each system below a) identify all critical points, b) construct the linear system for each critical point, and c) classify the critical points of the linear system according to stability (stable, unstable, asymptotically stable) and type (spiral, proper node, etc).

\[
\frac{dx}{dt} = x - 0.5xy, \quad \frac{dy}{dt} = -0.75y + 0.25xy
\]
A phase portrait of the system in the previous example is below.

We can also obtain the portrait using WolframAlpha:

\[
\text{streamplot}\left[\{x - 0.5xy, -0.75y + 0.25xy\}, \{x, 0, 4\}, \{y, 0, 4\}\right]
\]
Remaining Questions

From here, there are many questions that we could explore.

- How can we define and improve the accuracy of our model?
- How can we refine our model so that the prey obeys logistic growth in the absence of a predator?
- How can we determine if the solutions are periodic?

Many more details about non-linear systems in remaining sections of Chapter 7, other math courses like Math 4541.