Differences Between Linear and Nonlinear Equations (2.4)

1. Determine (without solving the problem) an interval in which the solution of the given initial value problem is certain to exist.

   a. \((t - 3)y' + (\ln t)y = 2t, \ y(1) = 2\)

   solution. Rewrite the equation into the standard form \(y' + p(t)y = g(t)\) by dividing the equation by \((t - 3)\). The interval that both \(p(t)\) and \(g(t)\) are continuous on and includes the initial condition is \((0, 3)\).

   b. \(y' + (\tan t)y = \sin t, \ y(\pi) = 0\)

   solution. The interval that both \(p(t)\) and \(g(t)\) are continuous on and includes the initial condition is \((\frac{\pi}{2}, \frac{3\pi}{2})\).

2. Given the fact that \(y_p = -5e^{3\sin 2x}\) is a particular solution of a homogeneous linear equation \(y' = f(x)y\), can you find all solutions of this differential equation?

   solution. Yes, we can find all the solutions. The general solutions of a homogeneous equation is of the form \(y = Cy_0\), where \(y_0\) is a nonzero particular solution. Thus, the general solutions are \(y = Ce^{3\sin 2x}\).

3. Given the fact that \(y_1 = e^x\) and \(y_2 = -2e^{-x}\) both satisfy the same nonhomogeneous linear equation \(y' = f(x)y + p(x)\), can you find all solutions of the nonhomogeneous linear equation \(y' = f(x)y + p(x)\)?

   solution. Yes, we can. Note that \(y_1 - y_2\) is a solution to the homogeneous equation. Therefore, the general solutions can be written as \(y = C(e^x - (-2e^{-x})) + e^x\).

For a more detailed solution to this problem, please see “Worksheet-ch2-solutions” under “2016Sring-2018Fall”.