Exam 1: Practice problems

1. (a) Write down the system of three ODEs describing the SIR transmission model.
(b) State five main model assumptions.
(c) Find three conserved quantities for the SIR model.

2. Consider the SIS transmission model where recovery results in immediate susceptibility.
(a) Write down the system of two ODEs describing the SIS transmission model.
(b) Find a conserved quantity.
(c) Use the conserved quantity to convert the system to a first order ODE $\dot{I} = f(I)$ and solve the resulting ODE.

3. Solve the first order ODE $2xy' + y = 6x$, $x > 0$ and $x(1) = 2$.

4. Consider the ODE $x' = rx + x^2$. Sketch all the qualitatively different velocity vector fields. Should a bifurcation occur, describe it in words and sketch the bifurcation diagram.

5. The growth of some tumors can be modeled by the law $\dot{N} = f(N) = -aN \log(bN)$ where $a, b > 0$, $f(0) = 0$, and $N(t)$ is proportional to the number of cells in the tumor. Determine the equilibrium points and their stability. Sketch the velocity vector field and the graphs of $N(t)$ for various initial values.

6. Carefully state the fundamental existence and uniqueness theorem for solutions of the IVP $\dot{x} = f(x)$, $x(t_0) = x_0$. 