Homework due and reading quiz on 01/31/19

1. Read Chapter 3 (Interferon) and Chapter 4 (Influenza) of Sompayrac

2. A very simple model of virus population dynamics is the ODE

\[
\frac{dV}{dt} = \lambda - PV,
\]

where \( V(t) \) is the virus population at time \( t \), the constant \( \lambda \) is the rate of virus production and \( P \) is the virus clearance rate.

(a) Find the equilibrium points and use the Method of Linearization to determine their stability.
(b) Find a formula solution of the linear ODE (You learned how to do this in the first week of ODEs, but you may use WolframAlpha, Mathematica, Matlab, etc.)
(c) Let \( V(0) = V_0 \). Find \( \lim_{t \to \infty} V(t) \).
(d) What happens to the long term limit when \( \lambda \) increases? When \( P \) increases?

3. A simple model for the population density of T-cells is

\[
\frac{dT}{dt} = s - dT + aT(1 - T/T_{\text{max}}).
\]

Here \( s \) is the rate at which new T-cells are produced, \( d \) is their death rate, \( a \) is the maximum proliferation rate, and \( T_{\text{max}} \) is the cell density at which the proliferation stops.

(a) Nondimensionalize this ODE.
(b) Find the equilibrium points.

4. A simplified version of the classic infection model assumes that the virus density is at equilibrium.

(a) Show that the resulting (nondimensionalized) ODE can be written as

\[
\begin{align*}
\frac{dx}{d\tau} &= 1 - x - R_0xy \\
\frac{dy}{d\tau} &= R_0xy - y
\end{align*}
\]

(b) Find the equilibrium points.
(c) Compare these equilibrium points with those of the full classical model.

5. A particular two species system can be modeled by

\[
\begin{align*}
\frac{dP}{dt} &= r_1P(1 - P/K) - A\frac{PQ}{D + P} \\
\frac{dQ}{dt} &= r_2Q\left(1 - \frac{Q}{P}\right)
\end{align*}
\]

(a) Describe how these two species interact.
(b) Nondimensionalize this system of ODE.
(c) Find the equilibrium points.