Homework due and reading quiz on 01/31/19

1. Read Chapter 3 (Interferon) and Chapter 4 (Influenza) of Sompayrac

2. A very simple model of virus population dynamics is the ODE

$$\frac{dV}{dt} = \lambda - PV,$$

where $V(t)$ is the virus population at time $t$, the constant $P$ is the rate of virus production and $c$ is the virus clearance rate.

(a) Find the equilibrium points and use the Method of Linearization to determine their stability.
(b) Find a formula solution of the linear ODE (You learned how to do this in the first week of ODEs, but you may use WolframAlpha, Mathematica, Matlab, etc.)
(c) Let $V(0) = V_0$. Find $\lim_{t \to \infty} V(t)$.
(d) What happens to the long term limit when increase $\lambda$? Increase $c$?

3. A simple model for the population density of T-cells is

$$\frac{dT}{dt} = s - dT + aT(1 - T/T_{\text{max}}).$$

Here $s$ is the rate at which new T-cells are produced, $s$ is their death rate, $a$ is the maximum proliferation rate, and $T_{\text{max}}$ is the cell density at which the proliferation stops.

(a) Nondimensionalize this ODE.
(b) Find the equilibrium points and use the Method of Linearization to determine their stability.
(c) Suppose $T_{\text{max}} = 1200 \text{ mm}^3/\text{day}, a = 0.5/\text{day}, d = 0.01/\text{day}$ and $s = 5.0/\text{day mm}^3$. Calculate the two equilibrium cell densities.

4. A simplified version of the classic infection model assumes that the virus density is at equilibrium.

(a) Show that the resulting (nondimensionalized) ODE can be written as

$$\frac{dx}{d\tau} = 1 - x - R_0 xy$$

$$\frac{dy}{d\tau} = R_0 xy - y$$

(b) Find the equilibrium points and use the Method of Linearization to determine their stability.
(c) Compare these equilibrium points with those of the full classical model.

5. More problems TBA